THE BOOK WAS DRENCHED

RIDERS IN EUCLID

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CONTAINING

A GRADUATED COLLECTION OF EASY DEDUCTIONS FROM BOOKS I., II., III., IV., AND VI. OF EUCLID'S "ELEMENTS OF GEOMETRY"

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PREFACE

THESE Exercises are set out in what I call *The Vertical and Horizontal Arrangement.* They may be taken in the vertical order in which they stand, or the Examples numbered 1, 1, 1, may be taken first, and then 2, 2, 2, and so on. Thus of the six Examples in each of the sets numbered I. to XXX.—

Three are on Euclid, Book I. to Prop. 33;

Two are on Book I., 34 to end of Book II.;

One is on Book III. to Prop. 16.

Further, in each Exercise from I. to XXX., Example

1. Is on Euclid, Book I. to Prop. 19.
2. „ „ Prop. 20 to 26.
3. „ „ „ 27 to 33.
4. „ „ „ 34 to 48.
5. „ Book II.
6. „ „ III. to Prop. 16.

So by taking the *first* Example in each Exercise in succession, a graduated series of deductions from early propositions in Book I. will be obtained, and a similar result will follow from taking the *second* Examples in succession, and so on for the *third* and following Examples.

Next, observe that in the Exercises numbered XXXI. to L.—

1 and 2 are on Euclid, I. to Prop. 33.

3 and 4 „ I., 34 to end of Book II.

5 and 6 „ III., 17 to end.

Lastly, in Exercise LI. the 4th and 6th Books of Euclid are brought in, and in this and the succeeding Exercises

1 and 2 are on Euclid, Books I. and II.

3 is on Book III.

4 is on Book IV.

5 and 6 are on Book VI.

My intention has been to make the Examples progressive, those first given in illustration of each Book of Euclid being very easy, and those in subsequent Exercises increasing gradually in difficulty, with easy deductions interspersed among the later sets.

I shall be grateful for advice as to the transposition or omission of particular Examples.

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CAMBRIDGE,
May 29, 1896.

RIDERS IN EUCLID

Exercise I.

1. AC and BC are the equal sides of an isosceles triangle. CD is a straight line bisecting AB in D. Show that CD also bisects the angle ACB.

2. MN, the base of an isosceles triangle MON, is produced to any point P. Show that OP is greater than ON.

3. Prove that in any acute-angled triangle any two of the angles are together greater than the third.

4. Show that the diagonals of a square make with each of the sides an angle equal to half a right angle.

5. Prove that the square on a straight line is equal to four times the square on half the line.

6. If the diameter AB of a circle cut a chord CD at right angles, prove that the triangles ABC, ABD are equal in all respects.

Exercise II.

1. The straight line, drawn from the vertex of an isosceles triangle to bisect the vertical angle, also bisects the base at right angles.

2. Prove that any three sides of a quadrilateral figure are together greater than the fourth side.

3. Show that the straight line, which bisects the external vertical angle of an isosceles triangle, is parallel to the base.

4. If the straight line joining two opposite angular points of a parallelogram bisect the angles, the parallelogram has all its sides equal.

5. Prove that the squares on the diagonals of a rectangle are together equal to the sum of the squares on the four sides.

6. Through a given point within a given circle, which point is not the centre, draw a chord which shall be bisected in that point.

Exercise III.

1. If in a triangle the perpendicular from the vertex bisect the base, the triangle is isosceles.

2. Show that any side of a triangle is less than half the sum of the sides.

3. If the straight line, bisecting the exterior angle ACD of the triangle ABC, be parallel to AB, the triangle is isosceles.

4. Show that the diagonals of a parallelogram bisect each other.

5. If the diagonals of a quadrilateral figure are at right angles to each other, the sum of the squares on one pair of opposite sides shall be equal to the sum of the squares on the other pair.

6. Two chords of a circle, which cut a diameter of the circle in the same point, and make equal angles with the diameter, are equal.

Exercise IV.

1. A given angle BAC is bisected. If CA be produced to Q, and the angle BAQ be bisected, show that the two bisecting lines are at right angles to each other.

2. PQR is an equilateral triangle. RM and QN are drawn at right angles to PQ and PR. Show that RM and QN are equal.

3. Show that the exterior angles of a quadrilateral figure, made by producing the sides taken in order, are together equal to the sum of the interior angles.

4. Prove that the diagonals of a square bisect each other at right angles.

5. Describe a square which shall be equal to the sum of three given squares.

6. Show that the line joining the centres of two circles, which cut one another, is perpendicular to the line joining their points of intersection.

Exercise V.

1. If one of the four angles, made by the intersection of two straight lines, be a right angle, prove that each of the other three angles is a right angle.

2. If the same straight line bisect the base and the vertical angle of a triangle, show that the triangle is isosceles.

3. If the straight lines bisecting the angles at the base of an isosceles triangle be produced to meet, show that they will contain an angle equal to an exterior angle at the base of the triangle.

4. Prove that the diagonals of a rhombus bisect each other at right angles.

5. Prove that the square on the hypotenuse of an isosceles right-angled triangle is equal to four times the square on the perpendicular drawn from the right angle to the hypotenuse.

6. P is a point within a given circle, of which the centre is O. Describe a circle, having P for its centre, which shall touch the given circle, and which shall lie inside it.

Exercise VI.

1. The straight line AD divides the right angle BAC into any two parts, and AE, AF are drawn making the angles BAE, CAF equal to BAD, CAD respectively. Show that AE, AF are in the same straight line.

2. O is a point within a triangle ABC. Show that the sum of the straight lines OA, OB, OC is less than the sum of AB, BC, CA.

3. Prove that the interior angles of a hexagon are together equal to eight right angles.

4. If the opposite angles of a quadrilateral figure be equal, prove that the figure is a parallelogram.

5. ABC is a right-angled triangle, having the right angle at A. AD is drawn perpendicular to BC. Show that the square on AD is equal to the rectangle contained by BD, CD.

6. The diameters AB, CD of two equal circles, which intersect one another, are parallel, and AD cuts the straight line joining the centres in M. Show that M is the middle point of that line.

Exercise VII.

1. With the vertex A of an isosceles triangle as centre, a circle is described which cuts the equal sides BA, CA, produced through A, in D and E respectively. Prove that CD is equal to BE.

2. If the straight line AD bisect the angle at A of the triangle ABC, and BDE be drawn perpendicular to AD and meeting AC, or AC produced, in E, show that BD is equal to DE.

3. How many sides has the rectilinear figure, the sum of whose interior angles is double of the sum of its exterior angles?

4. If two straight lines bisect each other, show that the lines joining their extremities will form a parallelogram.

5. Show how to describe a square equal to one-half of a given square.

6. If a straight line be drawn to cut each of two concentric circles, show that the parts of the line intercepted between the two circumferences are equal to one another.

Exercise VIII.

1. From the same point there cannot be drawn more than two equal straight lines to meet a given straight line

2. *ABC* is an isosceles triangle, and *BAC* is a right angle. The angle *ACB* is bisected by *CD*, which meets *AB* in *D*. *DN* is drawn perpendicular to *BC*. Show that *DN* is equal to *AD*.

3. The side *BA* of the isosceles triangle *ABC*, of which *A* is the vertex, is produced to *D*, so that *AD* is equal to *AB*. *C* and *D* are joined. Show that *BCD* is a right angle.

4. If two opposite sides of a quadrilateral be equal to one another, and the two remaining sides be also equal to one another, the figure is a parallelogram.

5. *BAC* is a right-angled triangle, having the right angle at *A*. *D* is any point in *AC*, and *E* is any point in *AB*. Show that the sum of the squares on *CE* and *BD* is equal to the sum of the squares on *DE* and *BC*.

6. A circle passes through the angular points of the triangle *ABC*. If *BAC* be a right angle, show that the centre of the circle is the middle point of *BC*.

Exercise IX.

1. From any point E in the straight line CD two equal straight lines EA, EB are drawn, making equal angles with ED; and CA, DA, CB, DB are joined. Prove that the triangles ACD, BCD are equal in all respects.

2. Upon the base AB of a triangle ABC is described a quadrilateral figure $ADEB$, which is entirely within the triangle. Show that the sides AC, CB of the triangle are together greater than the sides AD, DE, EB of the quadrilateral.

3. If two exterior angles of a triangle be bisected by straight lines which meet in O, prove that the perpendiculars from O on the sides, or the sides produced, of the triangle are equal.

4. If one diagonal of a quadrilateral bisect the other diagonal, it divides the quadrilateral into two equal triangles.

5. Show that the sum of the squares on two unequal lines is greater than twice the rectangle contained by those lines.

6. From the extremity A of the diameter AB of a circle equal straight lines AC, AD are drawn, making the angles BAC, BAD on opposite sides of AB equal to one another. CD, produced if necessary, meets the circumference in E and F. Prove that CE is equal to DF.

Exercise X.

1. In the equal sides AB, AC of an isosceles triangle ABC, or in these sides produced, are taken points D and E, equidistant from A; and BE, CD intersect in F. Prove that the triangles BFC, DFE are isosceles.

2. The angle ACB of a triangle is bisected by CD, and ADK, drawn perpendicular to CD, meets BC in K. Prove that AD is equal to DK.

3. How many sides has an equiangular polygon, four of whose angles are together equal to seven right angles?

4. If P be a point in a side AB of a parallelogram $ABCD$, and PC, PD be joined, the triangles PAD, PBC are together equal to the triangle PDC.

5. Describe a square that shall be equal to the difference between two given and unequal squares.

6. Q is a given point within a given circle, whose centre is O. Describe a circle, having Q for its centre, which shall touch the given circle, and which shall lie outside of it.

Exercise XI.

1. *ABC*, *ABD* are two isosceles triangles on the same base. Prove that the angle between *CA* and *DA* both produced is equal to the angle between *CB* and *DB* both produced.

2. Show that if the straight lines, bisecting the angles *ABC*, *ACB* of the triangle *ABC*, meet in *O*, and *OA* be joined, it will bisect the angle *BAC*.

3. In the triangle *FDC*, if *FCD* be a right angle, and the angle *FDC* be double of the angle *CFD*, show that *FD* is double of *DC*.

4. Show that the line, joining the middle points of the opposite sides of a square, is at right angles to each of those sides.

5. Show that the sum of the squares on the lines, joining the angular points of a square to any point within it, is double the sum of the squares on the perpendiculars let fall from that point on the sides of the square.

6. The diameter of a circle is 30 inches in length, and a chord 24 inches in length is drawn in the circle. Find the distance of this chord from the centre.

Exercise XII.

1. ABC is a triangle. On CA, BC are described equilateral triangles CEA, BDC, on the sides, away from the triangle. Prove that BE and AD are equal.

2. Through an angle of a given triangle draw a straight line cutting the opposite side, such that perpendiculars upon the line from the other two angles shall be equal.

3. If two equal straight lines AB, CD cross one another, and AC be equal to BD, then shall AD be parallel to BC.

4. Show that the figure, formed by joining by straight lines the middle points of a square, taken in order, is also a square.

5. A point D is taken in the side BC of an equilateral triangle ABC. Show that if A, D be joined, the square on AD, together with the rectangle contained by BD, DC, is equal to the square on BC.

6. Prove that no chord of a circle drawn through the middle point of another chord can be shorter than that chord.

Exercise XIII.

1. Describe an isosceles triangle having each of its equal sides double of the base.

2. How many triangles having sides 5 feet and 6 feet in length can be formed so that the third side shall contain a whole number of feet?

3. Prove that the bisectors of two angles of a triangle can never be at right angles to each other.

4. *ABCD* is a square. In *AB* take any point *E*, and in *BC*, *CD*, *DA* respectively make *BF*, *CG*, *DH* each equal to *AE*. Prove that *EFGH* is a square.

5. If a straight line be divided into three parts, the square on the whole line is equal to the sum of the squares on the three parts together with twice the sum of the rectangles contained by each two of the parts.

6. Show that a line which bisects two parallel chords in a circle is also perpendicular to them.

Exercise XIV.

1. On a given base is described an isosceles triangle, whose vertical angle is one-half that of an equilateral triangle described on the same base and on the same side of it. Prove that the distance between their vertices is equal to their common base.

2. If one side of a triangle be bisected, the sum of the other two sides is more than double of the line joining the vertex and the point of bisection.

3. From a given point, outside a given straight line, draw a line making with the given line an angle equal to a given rectilineal angle.

4. Show how to describe a square which shall be five times as great as a given square.

5. Show that the sum of the squares described upon the four sides of a rhombus is equal to the sum of the squares described upon the two diagonals.

6. If from a point within a circle two straight lines are drawn to the circumference equal to one another, the centre of the circle lies in the straight line bisecting the angle between the two lines.

Exercise XV.

1. The straight line OC bisects the angle AOB. Prove that if OD be any other straight line through O, without the angle AOB, the angles DOA, DOB are together double of the angle DOC.

2. If a point O be taken within an equilateral triangle ABC, such that the angle OAB is greater than OAC, then will the angle OCB be greater than OBC.

3. If the side BC of the triangle ABC be produced to D, and AE be drawn bisecting the angle BAC and meeting BC in E, show that the angles ABD, ACD are together double of the angle AED.

4. $ABCD$ is a parallelogram; E is the middle point of BC; E is joined to A and D, and AE, DC are produced to meet in F. Show that the triangle DEF is half the parallelogram $ABCD$.

5. If one of the acute angles of a right-angled isosceles triangle be bisected, the opposite side will be divided by the bisecting line into two parts, such that the square on one will be double of the square on the other.

6. If two circles cut one another, show that a line drawn through a point of intersection, terminated by the circumferences and parallel to the line joining the centres, is double of the line joining the centres.

Exercise XVI.

1. ABC is an isosceles triangle, AB being equal to AC. D is a point in AB, and E is a point in AC produced; and the straight line DE is bisected by BC. Prove that AD and AE are together equal to AB and AC together.

2. AOB is a straight line, and on the same side of it OP, OP' are drawn perpendicular to each other and of equal length. PM and $P'M'$ are drawn at right angles to AOB, meeting it in M and M'. Show that PM is equal to OM', and that $P'M'$ is equal to OM.

3. C is the middle point of a straight line AB. Prove that the sum of the perpendiculars from A and B on any line, which does not intersect the finite line AB, is double of the perpendicular from C on the same line.

4. If one angle of a rhombus be equal to two-thirds of a right angle, show that the diagonal drawn from the adjacent angular point divides the rhombus into two equilateral triangles.

5. Show, by means of Euclid II., 14, that the perimeter of a rectangle is greater than that of the square which is equal to the rectangle.

6. Show that the middle points of all chords of a circle, which pass through a given point within the circle, lie on the circumference of a certain circle.

Exercise XVII.

1. A given straight line AB is bisected in C, and on AC, BC are described equilateral triangles ACP, BCQ towards the same parts. PQ is joined and bisected in O. Show that the line joining O, C is perpendicular to AB.

2. AB, AC are straight lines meeting in A, and D is a given point. Draw through D a straight line cutting off equal parts from AB, AC, produced if necessary.

3. The triangle ABC has all its sides of unequal length. A line bisecting the angle BAC divides BC into two segments. Prove that the segment adjacent to the greater side of the triangle is greater than the other segment.

4. Prove that straight lines, bisecting two adjacent angles of a parallelogram, intersect at right angles.

5. The longer side AB of a rectangle $ABCD$ is produced to E, so that BE is equal to the shorter side BC. A circle is described on AE as diameter, and CB meets the circumference in F. If O be the centre of the circle, and FB be double of OB, prove that FC is equal to AB.

6. Through a given point within a given circle draw the least possible chord.

Exercise XVIII.

1. If points P, Q, R be taken in the sides AB, BC, CA of an equilateral triangle, such that AP, BQ, CR are all equal, show that P, Q, R will be the angular points of another equilateral triangle.

2. Prove that the sum of the distances of any point from the three angles of a triangle is greater than half the perimeter of the triangle.

3. If each of the equal angles of an isosceles triangle be equal to one-fourth of the vertical angle, and from one of them a perpendicular be drawn to the base, meeting the opposite side produced, then will the part produced, the perpendicular, and the remaining side form an equilateral triangle.

4. $ABCD$ and $DEFG$ are squares placed so that DC and DE are in the same straight line. Show that the diagonals AC and EG are parallel.

5. AB, AC are the two equal sides of an isosceles triangle. From B, BD is drawn perpendicular to AC, meeting it in D. Show that the square on BD is greater than the square on CD by twice the rectangle AD, CD.

6. Through one of the points of intersection of two equal circles, which cut one another, draw a straight line which shall be terminated by the circumferences and be bisected at the point of intersection.

Exercise XIX.

1. If AB, BC be the equal sides of an isosceles triangle, and if the circles whose diameters are AB, AC meet in D, prove that AD bisects the angle BAC.

2. ABC is a triangle, and the angle at A is bisected by a straight line which meets BC in D. Show that BA is greater than BD, and CA greater than CD.

3. The angle, between the bisector of the angle BAC of the triangle ABC, and the perpendicular from A on BC, is equal to half the difference between the angles at B and C.

4. If the sides of a triangle, taken in order, be produced to twice their original lengths, and the outer extremities be joined, the triangle so formed will be seven times the original triangle.

5. Prove that the sum of the squares on any two sides of a triangle is equal to twice the sum of the squares on half the base and on the line joining the vertical angle with the middle point of the base.

6. If two equal chords be drawn in a circle, and another chord be drawn through their middle points, the portions of this last chord intercepted between the middle points and the circumference are equal.

Exercise XX.

1. The lines drawn from the angular points of an equilateral triangle to the middle points of the opposite sides are equal.

2. The bisector of the exterior angle A of a triangle ABC meets the side BC produced in D. Prove that the perpendiculars drawn from D to the sides AB and AC, or these produced, are equal to one another.

3. Show how to trisect a right angle.

4. The side AB of the parallelogram $ABCD$ is bisected in E. CE is joined and produced to meet DA in F. Prove that the triangle FDC is equal in area to the parallelogram.

5. Prove that if the complements of the parallelograms about the diagonal of a given parallelogram are squares, the given parallelogram is also a square, and equal to four times each of the complements.

6. A parallelogram $ABCD$ is inscribed in a circle. Show that the parallelogram is rectangular.

Exercise XXI.

1. Show that the perpendiculars drawn to the three sides of a triangle from the middle points of the sides meet in the same point.

2. ABC is a triangle having the angle ABC acute. In BA, or BA produced, find a point D such that BD and CD are equal.

3. ABC is an isosceles triangle. Find points P, Q in the equal sides AB, AC, such that PB, PQ, QC may all be equal.

4. AB, CD, EF are three equal and parallel straight lines. Show that the triangles ACE, BDF, formed by joining the extremities of the lines towards the same parts, are equal.

5. If A be the vertex of an obtuse-angled isosceles triangle ABC, and BD the perpendicular from B on CA produced, show that the square on BC is equal to twice the rectangle contained by CA, CD.

6. Two equal circles cut one another, and a line is drawn through one of the points of intersection. If this line be a diameter of one of the circles, and if the part of it intercepted by the other circle be equal to the common chord of the circles, prove that this chord is equal to the radius of either circle.

Exercise XXII.

1. In the base of an isosceles triangle, of which the equal sides are AB and AC, points D and E are taken such that BD is equal to CE. Prove that the angle BAD is equal to the angle CAE.

2. M is the middle point of the base BC of an isosceles triangle ABC, and N is a point in AC. Show that the difference between MB and MN is less than the difference between AB and AN.

3. ABC is an equilateral triangle. D, E are points in BC, CA respectively, such that BD is equal to CE. If AD, BE be joined and intersect in O, show that the angle AOB is twice an angle of the equilateral triangle.

4. ABC is a right-angled triangle with the right angle at B. BD is drawn bisecting the angle ABC and meeting AC in D. DE and DF are drawn at right angles to AB and BC respectively. Show that $EBFD$ is a square.

5. If from the vertex of an isosceles triangle a straight line be drawn to any point in the base, the square on this line is less than the square on one of the equal sides of the triangle by the rectangle contained by the segments of the base.

6. Two equal chords AC, BD intersect within a circle. Show that they divide each other into segments which are equal, each to each.

Exercise XXIII.

1. The equal sides PQ, PR of an isosceles triangle PQR are produced to points M and N, so that $PM=PN$. QN and RM intersect in O. Prove that $QO=RO$, and also that PO bisects the angle at P.

2. Perpendiculars BD, CE are drawn from the ends of the base BC of an isosceles triangle ABC to the opposite sides. Show that the triangle ADE is also isosceles.

3. BC is a given straight line. Describe on BC an equilateral triangle ABC; bisect the angles at B and C by straight lines meeting in D, and prove that lines drawn through D parallel to AB and AC will divide BC into three equal parts.

4. If A, B be points in one, and C, D points in another, of two parallel straight lines, and the lines AD, BC intersect in E, show that the triangles AEC, BED are equal.

5. The square on the side AB of a triangle is equal to four times the square on the side BC; also the square on BC is equal to the difference of the squares on AB and AC. Find the magnitude of each of the angles of the triangle.

6. Two circles intersect in A and B, and they are met by a line parallel to AB in C, D, E, F. Prove that CD is equal to EF.

Exercise XXIV.

1. If AD, BE, CF be diameters of the circle ABC, prove that the triangles ABC, DEF are equal.

2. Two triangles ABC, BCD stand upon the same base, and on the same side of it, and they have the sides AB, CD equal, and also the sides AC, BD equal; these last two sides intersect in O. Prove that OA is equal to OD, and that OB is equal to OC.

3. ABC is a triangle. D, E are the middle points of AB, AC. EF is drawn parallel to AB and meeting BC in the point F. Prove that the triangles ADE, EFC are equal in all respects.

4. Prove that each of the parallelograms, which are about the diagonal of a rhombus, is a rhombus.

5. The lines drawn from the angular points of an equilateral triangle ABC to the middle points of the opposite sides meet in O. Prove that the squares on AO, BO, CO are together equal to the square on one of the sides of the triangle.

6. P is a point without a circle whose centre is O. PAB, PCD are drawn making equal angles with PO, and meeting the circumference in A, B and C, D respectively. Prove that AB and CD are equal.

Exercise XXV.

1. Construct an isosceles triangle having the sum of its equal sides equal to three times the base.

2. The sides BC, CA, AB of a triangle are bisected in D, E, F. Show that BC, CA, AB are together greater than AD, BE, CF.

3. $ABCD$ is any four-sided figure. Prove that the straight lines which bisect the angles at A and B, make with each other an angle equal to that between the lines which bisect the exterior angles at C and D.

4. OA, OB, OC are three equal straight lines, and the angles AOB, BOC, COA are also equal. Complete the parallelogram $AOBD$, and join O, D. Show that DOC is a straight line, and that OD is equal to OC.

5. ABC is a triangle. D, the middle point of BC, is joined to A. Prove that if the squares on BA, AC are together equal to the square on twice AD, the angle BAC is a right angle.

6. Two equal circles touch one another externally, and a parallelogram is formed by drawing two parallel diameters and joining their extremities. Show that the diagonals of the parallelogram pass through the point of contact of the circles.

Exercise XXVI.

1. On the circumference of a circle three points A, B, C are taken, so that the straight lines AB, BC are equal. Prove that the straight line joining B with the centre of the circle bisects the angle ABC, and cuts the straight line AC at right angles.

2. ABC is a triangle. D, E are two points in BC such that DE is one-half of BC. Show that the perimeter of the triangle ADE is greater than the sum of AB and AC.

3. The middle points of the sides of an equilateral triangle are joined to one another. Prove that the four triangles, into which these lines divide the equilateral triangle, are themselves equilateral.

4. If P and Q be two points in the diagonal AC of a parallelogram $ABCD$, and if PM, QL, drawn parallel to AB, meet BC, AD in M and L respectively, and if PK, QN, drawn parallel to AD, meet CD, AB in K and N respectively, prove that the parallelograms KL, MN are equal.

5. Given a square and one side of a rectangle which is equal to the square, find the other side.

6. Two equal chords of a circle cut each other, and two other chords are drawn bisecting the angles between the equal chords. Prove that one of these chords is bisected at the point of intersection by the other.

Exercise XXVII.

1. Find the point on a given straight line which is equidistant from two given points when they lie (1) on the same side of the line, and (2) on opposite sides of the line.

2. Show that the sum of the diagonals of a quadrilateral is less than the sum of any four lines that can be drawn from any point (except the intersection of the diagonals) to the four angular points.

3. AB, CD are two given straight lines. Through a point E between them draw a straight line GEH, such that the intercepted portion GH shall be bisected in E.

4. Two parallelograms $ABCD$, $ABEF$ are on the same base AB, and between the same parallels AB, DF. If AE, BC bisect each other in K, prove that the area $DABF$ is double of the parallelogram $ABCD$.

5. If ABC be a triangle having each of the angles B and C double of the angle A, and if BD bisect the angle ABC, and meet AC in D, the square on AD will be equal to the rectangle contained by AC and CD.

6. If from a point without a circle two equal straight lines be drawn to the circumference and produced, show that they will be at the same distance from the centre.

Exercise XXVIII.

1. ABC is an equilateral triangle. A point G is taken within the triangle, such that the angle GBC is equal to the angle GCB, and the line AG is drawn. Prove that AG bisects the angle BAC.

2. If the hypotenuse BC of a right-angled triangle ABC be produced to D, so that CD is equal to AB, then will AD be greater than BC.

3. ABC is a triangle. At C a straight line is drawn perpendicular to BC, meeting BA produced in a point D, such that AD is equal to AC. Prove that the triangle ABC is isosceles.

4. Two equal triangles stand on equal bases, in the same straight line, and on the same side of it. Show that the straight line joining the middle point of one of the sides of one of the triangles to the middle point of one of the sides of the other triangle is parallel to the straight line on which the bases lie.

5. Having given a straight line as the unit of measurement, draw straight lines whose measures are $\sqrt{2}$, $\sqrt{3}$, and $\sqrt{5}$, respectively. (NOTE.—This use of the word *Measure* is explained in Hamblin Smith's "Geometry," p. 95.)

6. A chord is drawn in a circle perpendicular to a diameter at a point midway between the centre and one end of the diameter. Prove that the ends of the chord and the other end of the diameter are the corners of an equilateral triangle.

Exercise XXIX.

1. With centre O a circle is described cutting a straight line in A and B. The angles OAB, OBA are bisected by straight lines meeting in C. Prove that AC, BC are equal to one another, and that OC produced will bisect AB.

2. $ABCD$ is a quadrilateral figure. The angles at A and B are bisected by lines meeting in O. Show that the angle AOB is equal to half the sum of the angles at C and D.

3. All the sides of a rectilineal figure are produced in order, and the interior angles of the figure are together equal to five times the sum of the exterior angles. How many sides has the figure?

4. Through A and C, the extremities of a diagonal of a parallelogram $ABCD$, straight lines AE and CE are drawn, the one parallel and the other perpendicular to the diagonal BD. If D and E be joined, prove that DE is equal to AB.

5. If the square on one of the sides containing the right angle in a right-angled triangle be three times the square on the other side, prove that the angle subtended by the first side is double of the angle subtended by the second side.

6. AB, CD are equal chords of a circle. Prove that AC parallel to BD, or AD parallel to BC.

Exercise XXX.

1. Two triangles ABC, DBC, equal in all respects, stand on opposite sides of the base BC. Prove that AD will either be at right angles to BC or will bisect it, and will, in either case, be bisected by BC.

2. A, B, C are three given points, not in the same straight line. Through A draw a line such that the perpendiculars upon it from B and C shall be equal.

3. A pentagon and a decagon are described, each having all its angles equal to one another. Prove that an angle of the pentagon is three-fourths of an angle of the decagon.

4. The diagonal AC of the parallelogram $ABCD$ is produced through A to E, so that AE is equal to AC, and upon AE and AB as adjacent sides the parallelogram $EABF$ is described. Prove that the diagonal AF is in the same straight line with, and equal to, AD.

5. The straight line AB is bisected at C and produced to D, and on CD is described a square $CEFD$. If the rectangle contained by AD and DB be equal to twice the square on BC, prove that the triangle ABE is equilateral.

[illegible]hree circles touch each other externally, and A, B, C are [illegible]ir centres. Show that the difference between AB [illegible] is half as great as the difference between the [illegible] the circles whose centres are B and C.

Exercise XXXI.

1. If in any triangle a line be drawn from the vertex to any point in the base, prove that the sum of the line so drawn and the base is greater than half the sum of the three sides of the triangle.

2. AB, AC are the equal sides of an isosceles triangle. D is a point in AB, and E is a point in AC produced, and the straight line DE is bisected by BC. Prove that AD and AE are together equal to AB and AC.

3. Through A and C lines are drawn parallel to the diagonal BD of a parallelogram $ABCD$, and through B and D lines are drawn parallel to AC. Prove that the parallelogram formed by these four lines is twice the parallelogram $ABCD$, and that its diagonals are double the sides of that parallelogram.

4. A straight line AB is divided into two unequal parts at C, and AC, CB are bisected in D and E. Prove that the difference of the squares on AE and BD is equal to three times the difference of the squares on CD and CE.

5. Show that the two tangents drawn from a given point to a circle are equal.

6. If two chords in a circle intersect, show that the triangles formed by joining their extremities are equiangular to each other.

Exercise XXXII.

1. Show that an exterior angle of a regular hexagon is equal to an interior angle of an equilateral triangle.

2. Show that the two perpendiculars drawn from the extremities of the side BC of a triangle ABC upon any straight line passing through A, are together not greater than twice the line drawn from A to the middle point of BC.

3. A line, not a diagonal, is drawn bisecting the parallelogram $ABCD$, and meeting AD, BC in E and F. Show that the triangles EBF, CED are equal.

4. If in a right-angled triangle the square on one of the sides containing the right angle be equal to three times the square on the other side, show that the two lines drawn from the right angle, respectively bisecting the hypotenuse and perpendicular to the hypotenuse, trisect the right angle.

5. If a quadrilateral figure $ABCD$ be described about a circle, show that the sum of AB and CD is equal to the sum of AD and BC.

6. If one side of a quadrilateral figure inscribed in a circle be produced, the exterior angle is equal to the opposite angle of the quadrilateral.

Exercise XXXIII.

1. Through each angular point of a triangle a straight line is drawn parallel to the opposite side. Prove that the triangle formed by these three straight lines is equiangular to the given triangle.

2. In a given triangle one of the angles is double of another. Prove that an isosceles triangle may be described on one of the sides of the triangle, such that the two triangles put together shall form an isosceles triangle.

3. $ABCD$ is a quadrilateral having BC parallel to AD. E is the middle point of DC. Show that the triangle AEB is half of the quadrilateral.

4. Describe a rectangle equal to a given square, and having the sum of two of its adjacent sides equal to a given straight line.

5. Two concentric circles being described, if a chord of the greater touch the less, the parts of the chord, intercepted between the two circles, are equal.

6. If the sides AB, DC of a quadrilateral inscribed in a circle be produced to meet in E, then will the triangles EBC, EAD have their angles equal, each to each.

Exercise XXXIV.

1. A point D is taken inside an isosceles triangle ABC, in which AB, AC are the equal sides, and the straight lines AD, BD, CD are drawn. If the angles DBC and DCB be equal to one another, prove that the triangles ADB, ADC are equal in all respects.

2. If two exterior angles of a triangle be bisected, and from the point of intersection of the bisecting lines a line be drawn to the opposite angle of the triangle, it will bisect it.

3. Describe a rhombus having one angle double of the adjacent angle, and having a given line for one of the diagonals.

4. Squares are described on the sides of a right-angled triangle. Lines equal to the diagonals of these squares are placed together so as to form a triangle. Prove that this triangle is right-angled.

5. Show that a circle cannot be described about a rhombus.

6. A is any point on a line PA touching a circle at P. If C is the centre of the circle, prove that the circle described on AC as diameter passes through P.

Exercise XXXV.

1. D is the middle point of the base BC of the triangle ABC. Show that the angle BAC is acute or obtuse, according as AD is greater or less than one half of BC.

2. If the sides AB, AC of a triangle be bisected in F and E, and if BE be equal to CF, show that the triangle is isosceles.

3. The areas of the four triangles into which a quadrilateral is divided by its two diagonals are all equal. Prove that the quadrilateral is a parallelogram.

4. The corners A and C of a quadrilateral figure $ABCD$ are equally distant from the middle point of the diagonal BD. Prove that the squares on AB and AD are together equal to the squares on CB and CD.

5. Describe a circle, the circumference of which shall pass through a given point, and touch a straight line in another given point.

6. Show that the lines bisecting any angle of a quadrilateral figure inscribed in a circle and the opposite exterior angle, meet in the circumference of the circle.

Exercise XXXVI.

1. In the sides AB, AC of a triangle ABC points D and E are taken, and the lines BE, CD, AF are drawn, F being the point of intersection of BE and CD. If BF and CF make equal angles with BC, and if AF bisects the angle DFE, show that the triangle ABC is isosceles.

2. Describe an isosceles triangle in which each of the angles at the base is one-fourth of the vertical angle.

3. If two opposite sides of a quadrilateral figure are parallel, and the other two sides are equal and not parallel, prove, that the angles adjacent to either of the first pair of sides are equal.

4. Points P and Q are taken on the sides BC, CD of a square $ABCD$, such that BP is equal to CQ, and AP, PQ, QA are joined. Show that the sum of the squares on the sides of the triangle APQ, together with eight times the triangle PCQ, is equal to four times the square $ABCD$.

5. From the external point A lines are drawn touching a circle at the points B, C; and the line joining A with the centre of the circle cuts the circumference in D. Show that BD bisects the angle ABC.

6. Show that the chords, which join the extremities of two diameters of a circle, form a rectangle.

Exercise XXXVII.

1. Describe a four-sided figure, all of whose sides shall be equal to each other and to one of the diagonals of the figure.

2. ABC is an isosceles triangle, whose vertex is A, and P is any point. If the angle PBC is greater than the angle PCB, show that the angle PAC will be greater than the angle PAB.

3. The sides AB, AC of a triangle ABC are bisected in D, E respectively. Prove that the triangle DBC is double of the triangle DEC.

4. Divide the hypotenuse of a right-angled triangle into two parts, such that the difference between the squares on these parts shall be equal to the square on the shortest side of the triangle.

5. Describe a circle touching two sides of a scalene triangle, or these produced, and having its centre on the third side.

6. AB, a chord of a circle, is the base of an isosceles triangle, whose vertex C is without the circle, and whose equal sides meet the circumference in D, E. Show that CD is equal to CE.

Exercise XXXVIII.

1. ABC is an equilateral triangle. D, E are points in BC, CA respectively, such that BD is equal to CE. If AD, BE be joined and intersect in O, show that the angle AOB is twice an angle of the equilateral triangle.

2. ABC is a triangle. The sides AB, AC are produced to D, E, so that BD and CE are each equal to BC. BE, CD intersect in O. Show that the angle BOD, together with half the angle BAC, is equal to a right angle.

3. Show that if two opposite sides of a quadrilateral be equal, and if the quadrilateral be bisected by one of its diameters, it will be a parallelogram.

4. Divide a given straight line so that the rectangle contained by the whole line and one part shall be equal to half the square on the other part.

5. Show that the tangents at the extremities of any chord of a circle make equal angles with the chord.

6. If in any quadrilateral the opposite angles be together equal to two right angles, show that a circle may be described about that quadrilateral.

Exercise XXXIX.

1. In the sides AB, AC of a triangle ABC points D, E are taken, and BE, CD are drawn meeting in F. Prove that the sum of DA, AE is greater than the sum of EF, FD.

2. ABC is any acute angle. AB is bisected in D, and at K in BC the angle DKB is made equal to the angle DBK. If AK be drawn, prove that it is perpendicular to BC.

3. If ABC be a triangle with the angle at A a right angle, and if O be the point of intersection of the diagonals of the square on BC, prove that AO bisects the angle BAC.

4. Describe a parallelogram equal to a given square, and having an angle equal to half a right angle, and one side equal to a given straight line longer than the side of the square.

5. Draw a tangent to a circle which shall be parallel to a given finite straight line.

6. Two circles, whose centres are A and B, touch each other externally at C. A straight line touches these circles at the points D and E, and DE is bisected in F. Prove that the angles of the triangle ABF are equal to those of the triangle CDE, each to each.

Exercise XL.

1. AB is a given straight line. C, D are given points on opposite sides of AB. Find a point P in AB such that the angles BPC, BPD are equal.

2. Find a point P in the base BC of a triangle ABC, such that if straight lines PD, PE be drawn through it and be terminated by the sides, the figure $PDAE$ may be a rhombus.

3. If CAB be a triangle, and O a point such that the triangles COA, COB are equal, show that CO or CO produced, bisects AB.

4. A straight line AB is divided at C so that the rectangle contained by AB, BC is equal to the square on AC. If from AC a part AD be cut off equal to BC, prove that AC is divided in D so that the rectangle contained by the whole line AC and one of its parts is equal to the square of the other part.

5. Two circles touch one another, and a straight line is drawn through the point of contact, meeting the circles again in the points AB. Prove that the straight lines touching the circles at these points are parallel.

6. O, O' are the centres of two circles PAB, QAB, which cut one another in A and B. Show that if O, O' lie on the same side of AB, and if AO (produced if necessary) meet the circle AQB in R, the angle ARO', OBO' are equal.

Exercise XLI.

1. Describe a rhombus having a given straight line as one of the diagonals, and each side double of that diagonal.

2. Points B, C are taken in the arc of a semicircle whose bounding diameter is AD. If B be nearer to A than C is, and if P be a point between D and the centre of the circle, prove that PB is greater than PC.

3. On the sides AB, AC of any triangle any parallelograms $ABDE$, $ACFG$ are described, situated outside the triangle. DE, FG are produced to meet in H, and AH is joined. On BC is described a parallelogram, having two of its sides equal and parallel to AH. Prove that this parallelogram is equal to the sum of the other two.

4. If in a quadrilateral the squares described on two opposite sides are together equal to the squares on the other two sides, prove that the diagonals of the figure are at right angles to each other.

5. If a tangent to two circles, which touch at C, meets them at A and B, show that ACB is a right angle.

6. If two opposite sides of a quadrilateral inscribed in a circle are parallel, show that the other two sides are equal, and also that the diagonals of the quadrilateral are equal.

Exercise XLII.

1. ABC, ABD, CDE, CDF are equilateral triangles. Show that EA, AB, BF are in the same straight line.

2. ABC is a triangle, right-angled at A. CD is drawn so that the angle ACD is equal to the angle ACB, and BE is drawn so that the angle ABE is equal to the angle ABC. Show that BE is parallel to CD.

3. E is a point on AB, the side of a square. Describe on AE a rectangle equal to the square.

4. From the angles of an acute-angled triangle BAC perpendiculars are drawn to the opposite sides, which meet in a point O. Show that the sum of the squares on OA, OB, OC is less than half the sum of the squares on the sides of the triangle.

5. A tangent to a circle meets the tangents at the extremity or a diameter in the points A and B. Show that AB subtends a right angle at the centre.

6. A circle is described about an equilateral triangle ABC, and the tangents drawn to the circle at the points A and B intersect in D. Prove that ABD is an equilateral triangle.

Exercise XLIII.

1. If from the extremity of the base of an isosceles triangle a line equal to one of the sides be drawn to meet the opposite side, the angle, formed by this line and the base produced, is equal to three times either of the equal angles of the triangle.

2. $ABCD$ is a rectangle, and AB is double of BC. On AB an equilateral triangle is constructed. Show that its area will be less than that of the rectangle.

3. In the sides BC, CD of a rhombus $ABCD$ points P, Q are taken, such that PQ is parallel to BD. Prove that the triangle ABP is equal to the triangle ADQ.

4. From any point O, within the acute-angled triangle ABC, perpendiculars are drawn meeting BC in D, CA in E, and AB in F. Show that the sum of the squares on AF, BD, CE is equal to the sum of the squares on FB, DC, EA.

5. Two circles intersect in A, B. Through A and B two lines CAD, EBF are drawn parallel to each other, meeting the circles in C, D and E, F. Show that $CEFD$ is a parallelogram.

6. Two circles, which meet in P, are touched at Q and R by two equal straight lines OQ, OR. Show that if OP touch one of the circles, it will touch both.

Exercise XLIV.

1. If an isosceles triangle can be divided into two isosceles triangles by a line drawn through one extremity of the base, show that its vertical angle is equal to two-fifths of a right angle.

2. Construct a right-angled triangle, of which the hypotenuse and one side are given.

3. Two equal triangles stand on the same base, and on the same side of it. If one side of one of the triangles bisect one side of the other triangle, show that the remaining sides of the triangles are parallel.

4. *ABC* is a straight line. From *B* a perpendicular *BD* is drawn to *AC*, so that the square on *AC* is equal to the sum of the squares on *AB*, *BC* and twice the square on *BD*. Prove that the angle *ADC* is a right angle.

5. Describe a circle of given radius to touch each of two given intersecting straight lines. How many such circles can be described?

6. *OA* and *OB* are drawn touching a circle in *A* and *B*. Show that the angle *AOB* is equal to the difference of the angles at the circumference subtended by and on opposite sides of *AB*.

Exercise XLV.

1. The sides of the triangle ABC are unequal. From A are drawn AD, AE meeting BC (or BC produced) at D and E. The angle BAD is equal to the angle ACB, and the angle CAE is equal to the angle ABC. Prove that the triangle ADE is isosceles.

2. Construct an isosceles triangle, the vertical angle of which shall be ten times either of the other angles.

3. Show that if a square and a rhombus are between the same parallels, one of the diagonals of the rhombus is greater and the other less than a diagonal of the square.

4. The diagonal BD of the square $ABCD$ is produced to E, the part produced being equal to a side of the square. Prove that the square on BE is equal to the rectangle contained by AC and AE.

5. Describe a circle of given radius to touch externally each of two given circles which intersect each other. How many such circles can be described?

6. An angle at the circumference of a circle is one-eighth of a right angle. What portion of the whole circumference is the arc on which the angle stands?

Exercise XLVI.

1. Construct a parallelogram, equal to a given triangle, and such that the sum of its sides shall be equal to the sum of the sides of the triangle.

2. If from the vertex A of a triangle ABC lines be drawn to meet the base in D and E in such a manner that the angles BAD and ACE are equal, and the angles CAE and ABD are also equal, then shall ADE be an isosceles triangle.

3. From a given point draw a straight line, which shall bisect a given parallelogram.

4. The line AB is divided into two unequal parts at C, and is produced in both directions to D and E respectively, so that AD is equal to AC, and BE to BC. Show that the difference of the squares on DB and EA is three times the difference of the squares on AC and BC.

5. Two circles touch internally at A, and BCD, a chord of the outer circle, touches the inner circle at C. Prove that CA bisects the angle BAD.

6. If the diameter of a circle be one of the equal sides of an isosceles triangle, show that the base will be bisected by the circumference.

Exercise XLVII.

1. $ABCDE$ is a five-sided figure. Prove that the straight lines AC, BD, CE, DA, EB are together greater than the five sides of the figure.

2. On the side BC of a square $ABCD$ an equilateral triangle BCE is drawn, both the square and the triangle being on the same side of BC. The diagonal DB of the square is produced through B to F. How many sides will a polygon have, if each of its angles be equal to the angle EBF?

3. Show that equal triangles between the same parallels are on equal bases.

4. The angle at A in the triangle ABC is a right angle. D and E are points in the side BC, and BD is equal to BA, and CE to CA. Prove that the square on DE is equal to twice the rectangle CD, BE.

5. If from a point without a circle, two tangents PT, PT', at right angles to one another, be drawn to touch the circle; and if from T any chord TQ be drawn, and from T' a perpendicular $T'M$ be dropped on TQ, then shall $T'M=QM$.

6. Through the extremities B, C of the base of the triangle ABC two circles are drawn, cutting AB, AC in D and E, D' and E' respectively. Prove that DE and $D'E'$ are parallel.

Exercise XLVIII.

1. Two houses, one on each of two intersecting straight roads, are equidistant from their junction. Show that any point which is equidistant from the houses, is also equidistant from the roads.

2. ABC is a triangle with unequal sides. The bisector of the angle at A meets BC in D. E is the middle point of BC. F is the foot of the perpendicular from A to BC. Show that AE, AD, AF are in order of magnitude.

3. $ABCD$ is a quadrilateral figure having AB parallel to and less than DC. Produce DC to E, so that the figure $ABED$ may be equal to twice the triangle DBC.

4. Given two sides of a triangle, snow that its area is greatest when they contain a right angle.

5. The two pairs of opposite sides of a quadrilateral inscribed in a circle are produced to meet. If the angles between these lines be bisected, show that the bisecting lines are at right angles to each other.

6. A chord CD is drawn at right angles to AB, a diameter of a given circle, and DP, another chord of the same circle, meets AB at the point Q. Prove that AC and BC are the internal and external bisectors of the angle PCQ.

Exercise XLIX.

1. The side BC of the triangle ABC is produced to D, and the angle ACB is bisected by a straight line which meets AB in E. Through E is drawn a straight line EF parallel to BC, meeting at F the line which bisects the angle ACD. Prove that EF is bisected at the point where it meets AC.

2. $ABCD$ is a parallelogram. On AB as base construct a triangle having one angle equal to DAB, and its area equal to that of the parallelogram.

3. If one trapezium has three of its sides equal to three of the sides of another trapezium, each to each, and the angles contained by the equal sides of the one equal to the angles contained by the sides equal to them of the other, then shall the figures be equal in all respects.

4. Inscribe a rhombus in a given triangle, so that one of its angles shall coincide with an angle of the triangle.

5. Two triangles ABC, BCD have the side BC common, the angles at B equal, and the angles ACB, BDC right angles. Show that the triangle ABC is double of the triangle BCD, if AB is double of BD.

6. If AD be drawn perpendicular to the side BC of a triangle ABC, and be produced to cut the circle ABC in N; then taking a point O in AD, such that OD is equal to DN, show that BO, CO are perpendicular to AC, AB respectively.

Exercise L.

1. Show how to find that point in a straight line the sum of whose distances from two points not in the line is the least possible.

2. Through the vertex of a triangle straight lines are drawn parallel to lines bisecting the angles at the base. Show that they will intercept on the base a straight line equal to the sum of the other two sides of the triangle.

3. A straight line is drawn cutting two adjacent sides of a given parallelogram at the points P and Q. Make a parallelogram which shall have one of its angular points in each side of the given parallelogram, and of which PQ shall be one side.

4. $ACDB$ is a straight line, and D is the middle point of CB. Prove that the square on AC is less than the sum of the squares on AD, DB by twice the rectangle AD, DB.

5. Two straight lines PAB, PCD are drawn from a point P to cut a circle $ABDC$. Show that, if from PB a length PE be cut off equal to PC, and if from PD a length PF be cut off equal to PA, the straight lines EF, BD are parallel.

6. The tangents at two points P and Q of a circle intersect in T. Prove that if R be any other point on the circumference, the chord of the circle drawn through T parallel to PR will be bisected by QR.

Exercise LI.

1. One diagonal of a quadrilateral figure bisects both of the angles through which it passes. Prove that the two diagonals of the figure are at right angles to each other.

2. A straight line AB is bisected at C and produced to D. Prove that the rectangle AC, AD is equal to the rectangle BC, BD together with twice the square on BC.

3. A, B, C are three points which divide the circumference of a circle into three equal parts, and P is any point on the circumference. Prove that the distance of P from one of the points A, B, C is equal to the sum of its distances from the other two.

4. If an equilateral triangle be inscribed in a circle, prove that the radii, drawn to the angular points, bisect the angles of the triangle.

5. Two equilateral triangles are drawn so that each side of either triangle cuts off a triangle from the corner of the other. Show that the six triangles so cut off are all similar.

6. P is a point in the diagonal BD of a rhombus $ABCD$. Show that the triangles APD, CPD have the same altitude.

Exercise LII.

1. If one angle of a parallelogram be bisected by the diagonal which passes through it, prove that all the other angles are also bisected by diagonals.

2. If A be the vertex of an acute-angled isosceles triangle ABC, and BD the perpendicular from B on AC, then the square on BC shall be equal to twice the rectangle AC, CD.

3. If two circles intersect in A and B, the tangents drawn to the circles from any point in AB produced are equal to one another.

4. Show that, in an equilateral triangle, the centre of the inscribed circle is equidistant from the three angular points.

5. Q is any point in the diagonal AC of the parallelogram $ABCD$. Show by means of Euclid VI. 1, that the triangles AQD, CQD are equal.

6. If ABC be any triangle, and BE, CF be the perpendiculars from the angular points B, C on the opposite sides, prove that AF is to AE as AC is to AB.

Exercise LIII.

1. Show that the two complements of any parallelogram are together not greater than one-half of the parallelogram.

2. A square is described on the hypotenuse of a right-angled triangle. From the intersection of the diagonals of the square perpendiculars are drawn on the other sides of the triangle. Show that these perpendiculars are equal.

3. From the point where two circles touch one another a straight line is drawn, cutting the circles at two other points. Prove that the tangents at these points are parallel.

4. If a circle be inscribed in a right-angled triangle, the difference between the hypotenuse and the sum of the other sides is equal to the diameter of the circle.

5. Show that the lines, drawn from the ends of the base of a triangle perpendicular to the line bisecting the vertical angle, are in the same ratio as the sides of the triangle.

6. P is the middle point of BC, a side of the triangle ABC. MN, drawn parallel to BC, meets AB, AC in M and N Show that the straight line AP bisects MN.

Exercise LIV.

1. *ABCD* is a quadrilateral figure, having *AB* parallel to and less than *DC*. Find a point *E* in *DC*, such that the triangle *DBE* may be equal to half the figure *ABCD*.

2. If one angle of a triangle be equal to the angle of an equilateral triangle, prove that the square on the side opposite to this angle, together with the rectangle contained by the other two sides, is equal to the sum of the squares on those two sides.

3. If a straight line touch a circle, and be parallel to a chord, the point of contact will be the middle point of the arc cut off by the chord.

4. *ABCDEFGH* is any eight-sided figure inscribed in a circle. Prove that the angles at *A*, *C*, *E*, *G* are together equal to the sum of the angles at *B*, *D*, *F*, *H*.

5. If the four sides of a quadrilateral figure be bisected, show that the lines joining the points of bisection, taken in order, will form a parallelogram.

6. *AB*, a chord of a circle, of which *AC* is a diameter, is produced to *M*, and *MP* is drawn at right angles to *AC* produced, meeting it in the point *P*. Show that $AB : AC = AP : AM$.

Exercise LV.

1. Given two angles of a triangle and the side adjacent to them, construct the triangle.

2. Produce a given straight line so that the square on the whole line thus produced may be double of the square on the given line.

3. If two chords of a circle, AEB, CED, intersect in E, show that the angles, subtended by AC and BD at the centre, are together double of the angle AEC.

4. Show how to describe a circle about a given rectangle.

5. P, Q are the middle points of AB, BC, sides of the triangle ABC. AQ and CP intersect in O, and PQ is drawn. Show that the triangles AOC, QOP are similar.

6. If, through any point in the diagonal of a parallelogram, a straight line be drawn, meeting two opposite sides of the figure, the segments of this line will have the same ratio as those of the diagonal.

Exercise LVI.

1 Find a point P in the hypotenuse AB of a right-angled triangle ABC, such that PB may be equal to the perpendicular from P on AC.

2. The triangles ABC, DBC are on the same base BC, and AD is parallel to BC. BD bisects AC in O. Prove that the triangles BOC, COD are equal.

3. Two circles, with centres A and B respectively, intersect in P. If PA be a tangent to the circle, whose centre is B, prove that BP is a tangent to the circle, whose centre is A.

4. The side of the equilateral triangle, described about a circle, is double of the side of the equilateral triangles, inscribed in the circle.

5. If D, E be points in the sides AB, AC respectively of the triangle ABC, such that the triangles DAC, EAB are equal, show that the sides AB, AC are divided proportionally in D and E.

6. Two straight lines are drawn, bisecting the angles at the base of an isosceles triangle. Show, by the application of Euclid VI. 3, that the straight line, joining the points in which they cut the sides, is parallel to the base.

Exercise LVII.

1. In the triangle ABC the angle at C is obtuse, and AD is drawn at right angles to BC produced. On AD produced F and G are taken such that DF is equal to AB, and DG is equal to AC. Prove that BG is equal to CF.

2. Make a parallelogram equal to a given parallelogram, and having its sides twice as long as those of that parallelogram.

3. Two circles touch each other, one of them being inside the other. Prove that any straight line drawn through their point of contact cuts off similar segments from the two circles.

4. Describe a circle having its centre on the base of a triangle and touching both the sides.

5. Two triangles ACB, ADB stand on the same base AB and on the same side of it. From a point E in AB lines are drawn parallel to AC, AD, meeting BC, BD in F and G. Show that FG is parallel to CD.

6. AB, AC are the equal sides of an isosceles triangle, and AD, BE are drawn perpendicular to BC, AC respectively. Prove that BE is to BC as AD to AB.

Exercise LVIII.

1. D is the middle point of the side BC of a triangle ABC, and P is any point on AD. Show that the triangles APB, APC are equal.

2. $ABCD$ is a parallelogram, and ABP, CDQ are equilateral triangles, external to the parallelogram. PA, QD meet in R, and PB, QC meet in S. Prove that $PRQS$ is a parallelogram, whose diagonals meet in the point in which the diagonals of $ABCD$ meet.

3. AB, CD intersect in O, and the rectangle contained by AO, OB is equal to the rectangle contained by CO, OD. Prove that a circle can be described about $ABCD$.

4. If $ABCDE$ be a regular pentagon inscribed in a circle, and BD, CE intersect at G, show that BG and EG are each equal to a side of the pentagon.

5. The lines AD, BE, drawn from the angular points A and B of any triangle ABC to the middle points D and E of the sides BC, AC, intersect in O. Prove that AO is twice as great as OD.

6. If a line touching two circles cut the line joining their centres, the segments of the latter will be to each other as the diameters of the circles.

Exercise LIX.

1. *ABCD* is a parallelogram, and *O* any point without it. Show that the difference of the triangles *OAB*, *OCD* is equal to half the parallelogram.

2. If an obtuse-angled triangle be isosceles, prove that the square on the side opposite to the obtuse angle is equal to twice the rectangle contained by one of the equal sides and the straight line made up of that equal side and the straight line intercepted without the triangle between the obtuse angle and the perpendicular on that side from the opposite angle.

3. Through any point *C*, in the common chord of two intersecting circles, a straight line *ABCDE* is drawn, cutting the circles in *A*, *D* and *B*, *E*. Prove that the rectangle *AC*, *CD* is equal to the rectangle *BC*, *CE*.

4. In any triangle the line bisecting an angle, and the line perpendicular to the opposite side at its middle point intersect on the circumference of the circumscribing circle.

5. *P* and *Q* are two points in the side *AB* of the triangle *ABC*, and *PM*, *QN* are drawn parallel to *BC*, meeting *AC* in *M* and *N*. Show that $BP : PQ = CM : MN$.

6. If, from the extremities of the diameter of a semicircle, perpendiculars be let fall on any line cutting the semicircle, the parts of the line intercepted between these perpendiculars and the circumference are equal.

Exercise LX.

1. The side BC of an equilateral triangle ABC is bisected at D, and the straight line DA is produced through A to a point E. From AE is cut off a part AF equal to AC, and CF is joined. How many sides will a polygon have, if each of its angles be equal to the angle CFE?

2. A straight line is bisected, and a rhombus is described on the middle segment. Show that the lines joining the corners of the rhombus to the adjacent extremities of the .trisected line are at right angles.

3. If tangents be drawn to a circle from any point without it, and a third line be drawn between the point and the centre of the circle, the perimeter of the triangle formed by the three tangents will be the same for all positions of the third point of contact.

4. Show that the square on a side of an equilateral triangle inscribed in a circle is equal to three-fourths of the square on the diameter of the circle.

5. Show how to cut off one-fifth of a given finite straight line.

6. If AD bisect the angle BAC of a triangle, meeting BC in D, and BE, CF be drawn parallel to AC, AB respectively, to meet AD in E and F, prove that DE is to AD as CD to DF.

Exercise LXI.

1. *BD*, *CD* the bisectors of the angles at *B* and *C* in the triangle *ABC* meet in *D*. Prove that, if angle *BDC* equals the sum of a right angle and a half right angle, then *BAC* is a right angle.

2. *ABCD* is a quadrilateral figure, and *O* is the middle point of *BD*. If the sum of the squares on *AB* and *AD* be equal to the sum of the squares on *BC* and *CD*, then *AO* is equal to *CO*.

3. Two tangents to a circle meet at a point, whose distance from the circle is equal to the radius. Prove that the tangents, together with the line joining their points of contact with their circle, form an equilateral triangle.

4. Describe a circle touching one side of a triangle and also touching the produced parts of the other two sides.

5. Show how to cut off three-fifths of a given finite straight line.

6. In the quadrilateral *ABDC*, *AB* is parallel to *CD*. Show that a straight line parallel to *AB* will cut *AC* and *BD*, or these produced, proportionally.

Exercise LXII.

1. In the side AD of a square $ABCD$ any point E is taken, and the side AB is produced to F, so that BF is equal to DE. Prove that EF is greater than a diagonal of the square.

2. A straight line AB is divided into two unequal parts at C, and AC and CB are bisected at D and E. Prove that the difference of the squares on AE and BD is equal to three times the difference of the squares on CD and CE.

3. If semicircles be described on the sides of a triangle which contain a right angle, show that they cut the hypotenuse in the same point.

4. Show that the square on the side of an equilateral triangle, inscribed in a circle, is triple of the square on the side of the regular hexagon, inscribed in the same circle.

5. AD is drawn bisecting BC, a side of the triangle ABC, in D. BEF is drawn bisecting AD in E, and meeting AC in F. Show that the triangles ABF and FBD are equal.

6. Show, by the application of Euclid VI. 3, that the bisectors of the angles of a triangle pass through the same point.

Exercise LXIII.

1. Prove that of all parallelograms, which can be formed with diagonals of given length, the rhombus is the greatest.

2. Through any point in the diagonel AC of a parallelogram $ABCD$ lines are drawn parallel to the sides meeting AB, BC, CD, DA in the points E, F, G, H. Prove that the triangles AEH, AFG are together equal to half the parallelogram $ABCD$.

3. ABC is a triangle. The line bisecting the angle BAC meets BC in D. A circle passes through A, touches BC in D, and cuts AB, AC in E, F. Prove that EF is parallel to BC.

4. In a right-angled triangle ABC a perpendicular AD is drawn from the right angle to the side BC. Show that AB touches the circle which circumscribes the triangle ACD.

5. Straight lines AE, BF are drawn at right angles to one another from the angular points A and B of a square $ABCD$, to meet CD, produced if necessary, at E and F. Prove that the rectangle contained by DE and CF is equal to the given square.

6. Two chords of a circle, AB and CD, are produced towards B and D so as to meet at E. CB is produced to meet at F a line through E parallel to AD. Prove that EF is a mean proportional between BF and CF.

Exercise LXIV.

1. If four straight lines be drawn bisecting the four angles of a rhomboid, prove that these straight lines will, by their intersection, form a rectangle.

2. On a given base construct an isosceles triangle equal to a given triangle on that base.

3. If two equal circles be drawn, each passing through the centre of the other, prove that the square on the common chord is three times the square on either radius.

4. Show that if the circle inscribed in a triangle touch two sides of the triangle at their middle points, it will touch the third side at its middle point.

5. AB is the diameter of a circle, and through A any straight line is drawn to cut the circumference in C, and the tangent at B in D. Show that AC is a third proportional to AD and AB.

6. Through the ends of the base BC of an equilateral triangle ABC, straight lines, drawn respectively perpendicular to AB, AC, meet in D. Prove that the area of the triangle BCD is one-third of the area of the triangle ABC.

Exercise LXV.

1. Upon the diagonal *BC* of a square *BDCE* a triangle *BAC* is described, having a right angle at *A*. Show that *AD*, *AE* respectively bisect the interior and exterior angles of the triangle at *A*.

2. If *ABC* be an isosceles triangle, and *DE* be drawn parallel to *BC*, cutting *AB* in *D* and *AC* in *E*, and *EB* be joined, show that the square on *BE* is equal to the rectangle *BC*, *DE* together with the square on *CE*.

3. If *O* be a fixed point outside a given circle, find a straight line such that each of the tangents drawn from any point in that line to the given circle shall be equal to the straight line joining that point to *O*.

4. *ABCDEF* is a regular hexagon. Prove that *AC* is equal to *AE*, also that *BF* is equal to *CE*, and that *AD* is perpendicular to *CE*.

5. If a straight line be drawn through the points of bisection of any two sides of a triangle, it will divide the triangle into parts which are to one another as 1 : 3.

6. *AB* is a diameter of a circle, and through *A* any straight line is drawn to cut the circumference in *C*, and the tangent at *B* in *D*. Show that *AC* is a third proportional to *AD* and *AB*.

Exercise LXVI.

1. The side BC of a triangle ABC is bisected in D, AD is bisected in E, and CE produced cuts AB in F. Prove that one of the parts into which AB is thus divided is equal to twice the other.

2. Show that in a right-angled triangle the square on the difference of the sides including the right angle is less than the square on the hypotenuse by four times the area of the triangle.

3. In the arc of a semicircle, whose bounding diameter is AD, points B and C are taken. AB and DC are produced to meet at the point E. AC and DB meet at F. Prove that if EF be joined it will be at right angles to AD.

4. A hexagon inscribed in a circle has a pair of opposite angles equal to one another. Prove that it has a pair of opposite sides parallel to one another.

5. Show that if $ABCDEF$ be a regular hexagon inscribed in a circle, the squares on AB, AC, AD are as $1 : 3 : 4$.

6. The side BC of a triangle ABC is trisected in D, E. DF, EG, drawn parallel to AB, meet AC in F, G. Show that the quadrilateral $DEFG$ is equal to the triangle ADE.

Exercise LXVII.

1. $ABCD$ is a parallelogram, and E a point within it. Prove that the sum of the areas of the triangles AEB and CED is independent of the position of E.

2. $ABCD$ is a square, and lines OE, OF, drawn parallel to the sides to meet AB in E and BC in F, are such that the figure $EBFO$ is a square. Prove that the point O lies on the diagonal DB.

3. From a point A, without a circle whose centre is O, two straight lines ABC, ADE are drawn cutting the circle. Prove that the difference between the angles COE and BOD is equal to twice the angle at A.

4. If the points of bisection of every pair of adjacent sides of a regular hexagon inscribed in a circle be joined, prove that the joining lines will form a regular hexagon.

5. If the side AB of the triangle ABC be bisected in D, and DE, DF be drawn bisecting the angles ADC, BDC, and meeting AC, BC in E, F respectively, show that EF is parallel to AB.

6. Two triangles ABC, DBC stand on opposite sides of the same base BC, and their vertices are joined by a straight line. Show that the triangles are as the parts of this line intercepted between the vertices and the base.

Exercise LXVIII.

1. In a triangle ABC, AD is drawn meeting BC at D, so that AC is equal to CD, and AE is drawn bisecting the angle DAC. If the angle AEC is equal to the angle BAC, prove that AD is equal to BD.

2. The straight line AB is bisected in C and produced to D. Show that if the square on AD be three times the square on CD, the square on BC is equal to twice the rectangle contained by BD, DC.

3. PMT is a tangent to the circle APC at the point P; $CNAT$ is a diameter, to which PN is drawn perpendicularly; and AM is perpendicular to PT. Prove that AM is equal to AN.

4. A circle passes through the four angular points of a rectangle. From any point in the circumference perpendiculars are drawn upon the four sides of the rectangle. Prove that the sum of the squares on these perpendiculars is equal to the square on the diagonal of the rectangle.

5. ABC is a triangle inscribed in a circle. Through B draw BD, parallel to the tangent to the circle at A, meeting AC, produced if necessary, in the point D. Prove that AB is a mean proportional between AC and AD.

6. If in the side BC of a parallelogram $ABCD$ a point E be taken, such that BE is one-fourth of BC, and if AE, BD be joined, meeting in F, show that BF is one-fifth of BD.

Exercise LXIX.

1. From the sides CB, BA, AC of an equilateral triangle equal lengths CR, BP, AQ are cut off. If the straight lines AR, BQ, CP intersect one another in the points L, M, N, prove that the triangle LMN is equilateral.

2. If the sides BC, CA, AB of a triangle ABC be produced to D, E, F so that $CD = 2BC$, $AE = 2CA$, $BF = 2AB$, prove that the area of the triangle DEF is nineteen times the area of the triangle ABC.

3. If the straight line, which bisects the vertical angle of a triangle, be produced to meet the circle described about the triangle, show that the tangent at the point of contact is parallel to the base of the triangle.

4. If O be the centre of the circle inscribed in a triangle ABC, and AO, BO be produced to meet the opposite sides in E and F, prove that if a circle can be described about the quadrilateral $CEOF$, the angle C is the third part of two right angles.

5. The base BC of a triangle ABC is bisected in D, and points E and F are taken in AB and AC respectively, such that AE and AF are equal. If AD and EF intersect in G, prove that EG is to FG as AC is to BA.

6. Prove that if a circle be described touching the side AB of a triangle ABC in B, and passing through the point C, and meeting AC, produced if necessary, in D, AD will be a third proportional to AC, AB.

Exercise LXX.

1. AB and ECD are two parallel straight lines. BF, DF are drawn parallel to AD, AE respectively. Prove that the triangles ABC, DEF are equal to one another.

2. Show that the area of any rectangle is equal to half the area of the rectangle contained by the diagonals of the squares upon two of its adjacent sides.

3. A straight line is drawn cutting off similar segments from two given circles. Prove that the four tangents at the points of intersection form, by their intersections, a parallelogram.

4. Show that the common chord of the two circles employed in the construction of Euclid IV. 10, is the side of a regular decagon inscribed in the circle.

5. If two circles touch each other internally, any two straight lines, drawn through the point of contact and terminated both ways by the circumferences, will be cut proportionally by the circumferences.

6. DE, a line parallel to BC, the base of the triangle ABC, meets AB and AC in the points D, E; CD and BE intersect in O. Show that AO produced bisects BC.

Exercise LXXI.

1. ABC is a triangle obtuse-angled at C. Straight lines are drawn perpendicular to the sides CA, CB at their middle points, cutting AB in D and E respectively, and D and E are joined to C. Prove that the angle DCE is equal to twice the excess of the angle ACB over a right angle.

2. Construct an isosceles triangle, having given the sum of the three sides and the sum of the vertical angle and one of the angles at the base.

3. A straight line AB is divided in C so that the rectangle AB, BC is equal to the square on AC, and on BC as base an isosceles triangle BCD is described, having its sides equal to AC. Prove that BD touches the circle which passes through A, C and D.

4. If with one of the angular points of a regular pentagon as centre and one of its diagonals as radius a circle be described, a side of the pentagon will be equal to a side of the regular decagon inscribed in the circle.

5. AE is drawn from the vertex A of an isosceles triangle perpendicular to the base BC. BD is a perpendicular from B on AC, cutting AE in F. Show that BF is to AC as BE to EA.

6. A tangent to a circle is drawn, and is terminated by two other parallel tangents. Prove that the radius of the circle is a mean proportional between the segments into which the first tangent is divided at the point of contact.

Exercise LXXII.

1. *ABCD* is a quadrilateral figure, such that *AB*, *AC*, *AD* are all equal. Show that the angle *BAD* is double of the angles *CBD* and *CDB* together.

2. Describe a square such that one of its angular points may be at one angle of an equilateral triangle, and the two sides not containing that angle may pass one through each of the other angular points of the triangle.

3. *A* and *B* are two given points inside a circle. Find a point *P* on the circumference, such that if *PA*, *PB* produced meet the circle again in *C* and *D*, the straight line *CD* may be parallel to *AB*.

4. Tangents are drawn to a circle at the extremity of an arc *AB*. Show that the centre of the circle which touches these tangents and the chord *AB* is at the middle point of the arc *AB*.

5. *ABC* is a triangle. At *A* a straight line *AD* is drawn, making the angle *CAD* equal to *CBA*, and at *C* the straight line *CD* is drawn, making the angle *ACD* equal to *BAC*. Show that *AD* is a fourth proportional to *AB*, *BC* and *CA*.

6. *ABCD* is a square. *P* and *Q* are points in *AB* and *DC* respectively, such that *AP* is to *PB* as 1 is to 4, and *DQ* is to *QC* as 2 is to 3. Find the ratios in which *PQ* cuts each of the diagonals.

Exercise LXXIII.

1. From the corners A, C of a parallelogram $ABCD$, AM and CN are drawn at right angles to BD; and from the corners B, D, BP and DQ are drawn at right angles to AC. Prove that $MQNP$ is a parallelogram.

2. Produce a given straight line AB to C, so that the rectangle contained by the sum and difference of AB and AC may be equal to a given square.

3. A is the centre of a given circle, and AB, AC are given straight lines. If with any point O on the circumference of the given circle as centre, and at distance OA, a circle be described cutting AB, AC in D and E, show that the straight line DE is of constant length.

4. Show how to inscribe an equilateral triangle in a given circle.

5. ABb, AcC are two given straight lines, cut by two other lines BC, bc so that the two triangles ABC, Abc are equal. Show that BC and bc divide each other in reciprocal proportion.

6. If CE be a third proportional to two straight lines AB, AC, show that half the sum of AB and CE is greater than AC.

Exercise LXXIV.

1. A line, parallel to the base BC of a triangle ABC, meets AB, AC in D and E. AF, AG are drawn, parallel to CD, BE respectively, to meet BC produced in F and G. Prove that CF is equal to BG.

2. Construct a triangle, having given the base, the sum of the sides, and one of the angles at the base.

3. AA', BB', CC' are equal arcs of a circle. AB, $A'B'$ meet in D, and BC, $B'C'$ meet in E. Prove that a circle will pass through B, B', D, E.

4. In a given triangle inscribe a rhombus, one of whose sides shall coincide in direction with a side of the triangle, and one of whose angles shall be at a given point in that side.

5. OA, OB are radii of a circle at right angles to one another. A line ACD is drawn through A meeting OB in C, and the circle in D. Prove that the rectangle contained by AC, AD is equal to the square on AB.

6. If two circles touch each other in a given point, and have also a common tangent which does not pass through that point, show that the part of the tangent between the points of contact is a mean proportional between the diameters of the circles.

Exercise LXXV.

1. If through O, a point between two parallel lines, a line is drawn having its extremities on the lines, and this line be bisected at O, then every line through O, having its extremities on the parallel lines, is bisected at O.

2. On the diagonal of a parallelogram describe a rhombus equal to the parallelogram.

3. Find a point within a given triangle such that each side subtends at it the same angle. Is there necessarily such a point?

4. A circle circumscribes a rectangle $ABCD$. Another circle is described with centre A and radius AB, cutting the first-named circle at E. Prove that CE is equal to AD, and that DE is parallel to AC.

5. Two circles cut each other at A and B, and CAD is any common chord of the circles. Prove that BC and BD are to each other as the diameters of the circles of which they are chords.

6. The bisector of the angle BAC of a triangle meets BC in D, and meets in E the straight line which bisects BC at right angles. Prove that the rectangle contained by ED and EA is equal to the square on EB.

Exercise LXXVI.

1. If either diagonal of a parallelogram be equal to a side of the figure, show that the other diagonal is greater than any side of the figure.

2. A straight line is divided into two parts, such that one of them is equal to three times the other. Show that the rectangle contained by these parts is three-sixteenths of the square on the whole line.

3. O is the centre of a given circle, and A is a given point in its circumference. Find a point in OA produced, such that the two tangents drawn from that point to the circle shall make with each other an angle equal to a given angle.

4. Show that the tangents drawn from the angular points of a triangle to a circle, concentric with and within the circle described about the triangle, are equal.

5. The angle C of a triangle ABC is bisected by a line which cuts AB in F. The angle B is bisected by a line which cuts CF in G. Prove that AF is to FG as AC to CG.

6. Divide a circle into two segments, such that the angle contained by one of the segments shall be double of the angle contained by the other.

Exercise LXXVII.

1. If two triangles have two sides respectively equal and the included angles supplementary, prove that the triangles are equal in area.

2. The hypotenuse AB of a right-angled triangle ABC is trisected in the points D, E. Prove that if CD, CE be joined, the sum of the squares on the sides of the triangle CDE is equal to two-thirds of the square on AB.

3. Two circles intersect in A and B. Through A any straight line is drawn, which meets the circles again in C, D. Prove that the angle CBD is constant, and equal to the angle subtended at B by the straight line joining the centres of the circles.

4. Upon a given straight line, as base, describe an isosceles triangle, having the angle at the vertex treble of each of the angles at the base.

5. A, B, C are three points in order in a straight line. Find a point P in the straight line such that PB may be a mean proportional between PA and PC.

6. In a quadrilateral figure $ABCD$, the side AB is parallel to the side CD, and CD is three times as great as AB. The diagonals AC, BD intersect in O. Prove that AO is equal to one-fourth of AC.

Exercise LXXVIII.

1. Bisect a given triangle by a straight line drawn from a given point in one of its sides.

2. Prove that a quadrilateral which has two opposite sides and two opposite obtuse angles equal, is a parallelogram. Show also that the figure is not necessarily a parallelogram, if the equal angles be acute.

3. Through a point A on the circumference of a circle two equal chords AB, AC are drawn. A chord AD, situated within the angle BAC, cuts the chord BC at E. Prove that AB touches the circle which circumscribes the triangle BDE.

4. From A, B, C, the angular points of a triangle, AD, BE, CF are drawn at right angles to the opposite sides, and they intersect in P. O is the centre of the circle ABC. Show that the centre of the circle DEF will bisect OP.

5. Two circles intersect at A and B, and at A tangents are drawn, one to each circle, to meet the circumferences at C and D. Show that if CB, BD are joined, BD is a third proportional to CB, BA.

6. $PQRS$ is a square, and X, Y are the middle points of the sides PS, RS respectively. Prove that the straight lines QX, QY trisect the diagonal PR.

Exercise LXXIX.

1. If a quadrilateral figure have two sides parallel, and the parallel sides be bisected, the line joining the points of bisection shall pass through the point in which the diagonals cut one another.

2. Divide a given straight line into two parts such that the squares on the whole line and on one of the parts shall be together double of the square on the other part.

3. If the tangent PT at any point P on a circle meet a diameter AB produced in T, and if P be joined to B, the extremity of the diameter nearer to T, show that the angle ATP together with twice the angle BPT make up a right angle.

4. $ABCDE$ is a regular pentagon inscribed in a circle. Join AC, BD, CE, DA, and show that these lines form by their intersections an equiangular pentagon.

5. The side BC of a triangle ABC is produced to D, making CD equal to BC. AB is bisected in E, and EB is joined cutting AC in F. Prove that EF is to FB as CF to AF.

6. If any triangle be inscribed in a circle, and from the vertex a line be drawn parallel to a tangent at either extremity of the base, this line will be a fourth proportional to the base and the two sides.

Exercise LXXX.

1. Trisect a parallelogram by straight lines drawn from one of its angular points.

2. If AB, one of the equal sides of an isosceles triangle ABC, be produced beyond the base to D, so that BD is equal to AB, show that the square on CD is equal to the square on AB together with twice the square on BC.

3. From a given point, without a given circle, draw a line to cut the circle, so that the part intercepted by the circle is three times the part without the circle. When is the problem impossible?

4. If a regular hexagon and an equilateral triangle be inscribed in the same circle, the sum of the squares on a side of each is equal to the square on the diameter of the circle.

5. If P be such a point that the triangles BAP, CAP are to one another as AB to AC, show that AP bisects the angle BAC when the points A, B, C are not in the same straight line.

6. On the side BC of an acute-angled triangle ABE a circle is described. On AB a point D is taken such that AD is equal to the tangent drawn from A to this circle, and DE is drawn at right angles to AB to meet AC produced in E. Prove that the triangle ADE is equal to the triangle ABC.

Exercise LXXXI.

1. Describe a rhombus equal to a given square, and having one of its diagonals double of the other.

2. Draw a straight line from an angular point of a triangle so as to cut off from the triangle a part equal to a given triangle.

3. From the point of contact of two circles, which touch one another, straight lines are drawn to the extremities of a diameter of one of the circles. Show that if these lines, produced if necessary, meet the other circle in P and Q, the line PQ will pass through the centre of the latter circle.

4. Inscribe a circle in a given rhombus.

5. If a line touching two circles cut another line joining their centres, the segments of the latter line will be to one another in the ratio of the diameters of the circles.

6. If from O perpendiculars OE, OF, OG, OH be drawn to the sides AB, BC, CD, DA of the parallelogram $ABCD$, prove that $EH : FG = OA : OC$.

Exercise LXXXII.

1. Given the two sides of a triangle, and the straight line drawn from the vertex to the middle point of the base, construct the triangle.

2. Describe a triangle equal to a given quadrilateral figure.

3. If through a fixed point within a given circle two chords be drawn at right angles to each other, show that the sum of the squares described on the four segments of the chords is constant.

4. A point P is taken in the side AB of a triangle ABC, and circles are described about the triangles APC, BPC. Show that the angle between the straight lines which touch these circles at P is independent of the position of P in AB.

5. A perpendicular is drawn from the right angle of a right-angled triangle to the base. Show that it divides the base into parts which are in the duplicate ratio of the adjacent sides.

6. Find a point D in the side AB of a triangle ABC, such that a line drawn through D, parallel to the base BC, will divide the triangle into two parts which will be in the ratio of AD to AB.

Exercise LXXXIII.

1. If the two diagonals of a parallelogram divide it into four triangles which are equal to one another in all respects, prove that the parallelogram is a square.

2. Show how to bisect a given quadrilateral figure by a straight line drawn through one of its angular points.

3. From an external point A draw a straight line ABC cutting a given circle in B, C, such that AC shall be bisected in B.

4. The sides AD, BC and AB, CD of a quadrilateral $ABCD$, inscribed in a circle whose centre is O, meet respectively in E and F. From E a perpendicular EM is let fall on OF. Prove that the rectangle OF, OM is equal to the square on the radius of the circle.

5. The points D and E are taken, on the sides AB and BC of the triangle ABC, such that AD and BE are each one-third of AB and BC respectively, and the lines AE and CD intersect in F. Prove that $AF : FE = 3 : 4$.

6. The diagonals of a quadrilateral figure $ABCD$ intersect at the point O. If the triangle ABC be double of the triangle ADC, prove that OB is equal to twice OD.

Exercise LXXXIV.

1. Draw a rhombus equal to a given triangle.

2. $ABCD$ is a square, and lines GOE, HOF, drawn parallel to the sides, meet AB in E, BC in F, CD in G, and DA in H. Prove that if the figure $OEBF$ is a square, so also is the figure $OGDH$.

3. $ABCD$ is a straight line divided so that AB is equal to CD, and circles, described with centres B and C at the distances AB, CD respectively, intersect in E. Show that AE touches the circle passing through the points B, D, E.

4. The bisector of the angle BAC of a triangle ABC meets the side BC in D. The circle described about the triangle ABD meets CA again in E, and the circle described about the triangle CAD meets BA again in F. Show that BF is equal to CE.

5. Through the angular points A, B, C of an equilateral triangle straight lines are drawn perpendicular to the sides BC, CA, AB respectively, so as to form another equilateral triangle. Compare the areas of the two triangles.

6. ADE is a straight line which divides the base of a triangle ABC so that BD is to DC as BA to AC, and which cuts in E the circle described about ABC. Show that the rectangle contained by AB, AC is equal to the difference of the squares on AE, BE.

Exercise LXXXV.

1. Through two given points draw two lines, forming with a line, given in position, an equilateral triangle.

2. AB is a straight line divided at E so that the square on AE is equal to the rectangle contained by AB, EB. $ABCD$ is a square, and DE is joined and produced to meet CB produced in F. Show that BF is equal to AE.

3. O and P are any two points on a circle. With centre O and any radius less than OP a circle is described so as to cut the first circle in A and B. Prove that PO bisects the angle APB.

4. If the smaller circle, employed in Euclid's Construction for IV. 10, cut the larger circle in D and E, show that BDE is an isosceles triangle, having the angle at D eight times as large as either of the other angles.

5. Find a point within a given triangle, such that two of the triangles formed by joining it to the angular points of the given triangle are each double of the third triangle.

6 AB, AC, sides of the triangle ABC, are bisected in D and E. Prove that the quadrilateral $DBCE$ is equal to three times the triangle ADE.

Exercise LXXXVI.

1. Show how to divide a given parallelogram into five triangles of equal area by straight lines drawn from one of its angular points.

2. ABC is a triangle having an obtuse angle at A. From the ends of BC perpendiculars BM, CN are drawn on the other sides produced. Show that the rectangle contained by AB, AN is equal to the rectangle contained by AC, AM.

3. ABC is an equilateral triangle. D, E, F are the middle points of BC, CA, AB respectively. Show that DF touches the circle passing through the points D, E, C.

4. Construct a triangle which shall have three given points for the middle points of its sides.

5. ABC is a triangle, and D is the middle point of BC. DE, DF are drawn bisecting the angles ADC, ADB, and meeting AC, AB in E and F. Prove that EF is parallel to BC.

6. $ABCD$ is a quadrilateral figure inscribed in a circle. BA, CD are produced to meet in P and BD, BC are produced to meet in Q. Prove that PC is to PB as QA is to QB.

Exercise LXXXVII.

1. *OAB*, *OAC* are two triangles having the base *OA* common, their vertices *B*, *C* on opposite sides of *OA*, and their areas equal. Prove that if lines *BD*, *CD* be drawn parallel to *AB*, *AC* respectively they will meet on *OA*, or *OA* produced.

2. On the two sides of a right-angled triangle, which contain the right angle, rectangles equal in all respects are described. Show that two of their diagonals are parallel, and that the other two lie in one straight line parallel to the hypotenuse of the triangle.

3. If *C* be the centre of a circle, *O* an external point, *OP* a tangent, *PN* perpendicular to *OC*, then the square on *OP* is equal to the rectangle *OC*, *ON*.

4. In a given square inscribe an equilateral triangle, so that one of its angular points may lie on a given angular point of the square, and the other two may lie one on each of the sides of the square not containing that angle.

5. *ABC*, *CDE* are equal triangles with equal angles at *C*, and they are on opposite sides of *BCE*, which is a straight line. Show that a straight line through *C*, parallel to *BD* and terminated by *AB* and *DE*, is bisected in *C*.

6. Prove that the regular octagon inscribed in a circle is a mean proportional between the square inscribed in the circle and the square circumscribing the circle.

Exercise LXXXVIII.

1. In the side BC of the equilateral triangle ABC any point D is taken, and the side AC is produced to E, making CE equal to BD. Show that the triangle ADE is isosceles.

2. A is a fixed point, and B is a fixed point in a given line BC. Find a point P in BC, such that the sum of the lengths of AP and BP may be equal to a given length.

3. One circle is wholly within another and contains the centres of both. Find the greatest and least chords of the outer circle touching the inner circle.

4. Having given the radius of a circle, find its centre, when it is known that the circle touches two given lines, which are not parallel.

5. Show how to bisect a triangle by a line drawn parallel to one of its sides.

6. If a quadrilateral figure be inscribed in a circle, and perpendiculars be drawn from the angular points upon the diagonals, show that the lines joining the feet of these perpendiculars will form a quadrilateral figure similar to the original figure.

Exercise LXXXIX.

1. The angle BAC of a triangle ABC is bisected by a straight line, which meets at F the straight line drawn through the middle point of AC parallel to AB. If CF be joined, prove that CF is at right angles to AF.

2. The magnitudes of A, B, C, the angles of a triangle ABC, are known. Find the magnitudes of the angles of the triangle formed by the lines bisecting the exterior angles of the given triangle.

3. On two sides of any triangle semicircles are described. Show that they cut the third side, or the third side produced, in the same point.

4. Inscribe a circle in a given regular octagon.

5. If the middle points of the diagonals of any quadrilateral be joined to the middle points of two opposite sides, show that the figure so formed is a parallelogram.

6. The diameter AB of a circle is produced towards A to any point P, and from P a tangent is drawn to the circle. From the point of contact a perpendicular is drawn upon AB, meeting it at M, and C is the centre of the circle. Prove that PA is to PC as PM is to PB.

Exercise XC.

1. Through the angular points A, B, C of a triangle are drawn three parallel lines meeting the opposite sides, or these produced, in P, Q, R respectively. Prove that the triangles AQR, BRP, CPQ are all equal.

2. If an angle of a triangle be twice as large as an angle of an equilateral triangle, show that the square on the side subtending that angle is equal to the sum of the squares on the sides containing it, together with the rectangle contained by those sides.

3. Two circles intersect at a point, and the tangents to the two circles at this point are perpendicular to each other. If the diameters of the circles which pass through this point be drawn, and their other extremities be joined, prove that the joining line will pass through the other point of intersection of the circles.

4. If the diagonals of a quadrilateral inscribed in a circle are at right angles to each other, show that the line, drawn from their point of intersection perpendicular to a side, will when produced bisect the opposite side.

5. Two chords AB, CD of a circle intersect at right angles within the circle. Show that the arcs AC, BD are together equal to half the circumference.

6. If the side BC of a triangle ABC be bisected by a straight line which meets AB and AC, produced if necessary, in D and E respectively, show that AE is to EC as AD to DB.

Exercise XCI.

1. In the side AC of any triangle ABC take any point D. Bisect AD, DC, AB, BC at the points E, F, G, H respectively. Show that EG is parallel and equal to FH.

2. From AC, the diagonal of a square $ABCD$, cut off AE equal to one-fourth of AC, and join BE, DE. Show that the figure $BADE$ is equal to twice the square on AE.

3. CD is drawn from the angle C perpendicular to the hypotenuse AB of a right-angled triangle ABC. DE, DF are drawn parallel to AC, BC, meeting BC, AC in E, F respectively. Show that a circle can be described about the figure $ABEF$.

4. ABC is a triangle inscribed in a circle. AD is drawn bisecting the angle BAC and meeting the circumference in D. If O be the centre of the circle inscribed in the triangle ABC, show that OD is equal to OB.

5. The circumference of one circle passes through the centre O of another circle, and through A, one of the points of intersection, a diameter AB of the first circle is drawn, meeting the other circle in C. Show that the rectangle AB, AC is equal to twice the square on CO.

6. Two circles intersect in P, Q, and the tangents at Q cut the circles in B, C respectively. Show that PQ is a mean proportional between BP and CP.

Exercise XCII.

1. The diagonals of a parallelogram intersect in O. Prove that any straight line KOL, drawn through O to meet the opposite sides, in K and L is bisected in O.

2. If a straight line, terminated by the sides of a triangle, be bisected, show that no other line terminated by the same two sides can be bisected in the same point.

3. AB is the tangent at A to a circle, and BP is drawn to meet the circle in P, so that PB produced will pass through the centre. Show that twice the angle PAB together with the angle BPA make up a right angle.

4. Construct a triangle, having given the three angles and the radius of the circle which can be inscribed in the triangle.

5. AB being a given straight line, find a point P in AB produced such that PA may be to PB in a given ratio.

6. Divide a given straight line so that the rectangle contained by the parts may be equal to a given triangle.

Exercise XCIII.

1. If the opposite angles of a hexagon *ABCDEF* are equal, that is the angles at *A*, *B*, *C* equal to the angles at *D*, *E*, *F* respectively, show that the opposite sides are parallel.

2. *P* and *Q* are the middle points of *AB*, *AC* sides of the triangle *ABC*. *CP*, *BQ* intersect in *O*. Show that the figure *APOQ* is equal to the triangle *BOC*.

3. A straight line *AB* is divided at any point *C*. On *AC* as diameter is described a circle, of which any chord *AP* is drawn. Show that if *AP* is produced to *Q*, so that the rectangle *AP*, *AQ* is equal to the rectangle *AB*, *AC*, the point *Q* lies on a fixed straight line.

4. Compare the areas of two regular hexagons, one inscribed in, and the other described about, a given circle.

5. *A* and *B* are the points of intersection of two circles, and a straight line *DAE* is drawn through *A*, meeting one circle in *D* and the other in *E*. Show that *DE* will be longest when it is at right angles to *AB*.

6. Cut off the third part of a triangle by a line drawn parallel to one of its sides.

Exercise XCIV.

1. Find the magnitudes of the angles of the triangle formed by joining the points in which perpendiculars from the angular points of a triangle, whose angles are known, meet the opposite sides.

2. Describe a rhombus equal to a given square, such that one of the diagonals of the rhombus shall be double of the other diagonal.

3. The tangents at B and C to a circle ABC, whose centre is D, intersect in O, and A is any point on the circumference. Prove that the angle BAC is equal to the angle ODC, and therefore that the chord of the circle parallel to BA, and passing through O, is bisected by AC.

4. Describe a parallelogram about a given circle, having one of its angles equal to a given angle.

5. ABC is the diameter of a circle, CD is a radius perpendicular to it. The chord AD is bisected in E. BE meets CD in F, and the circumference in G. Show that three times the rectangle contained by BF, EG is equal to the square on the radius.

6. Show how to inscribe in a given square another square, whose area is three-fourths of the area of the given square.

Exercise XCV.

1. From the sides AB, BC, CD, DA of a square equal lengths AP, BQ, CR, DS are cut off, and the straight lines AQ, BR, CS, DP intersect one another in the points L, M, N, O. Prove that the quadrilateral $LMNO$ is a square.

2. If two sides of a quadrilateral be parallel, the triangle bounded by either of the other sides and the two straight lines, drawn from its extremities to the middle point of the opposite side, is half the quadrilateral.

3. The arc BC of a circle $ABCD$ is double of the arc AB, and D is any point on the arc ADC. If P is the middle point of the arc BC, and if DA, CP produced meet the tangent at B in M, N respectively, prove that a circle can be described about the quadrilateral $DMNC$.

4. Given the base, the vertical angle, and the altitude of a triangle, construct the triangle.

5. Having given the base of a triangle, and the ratio of its sides, prove that the vertex lies on a certain circle.

6. The triangle ABC is right-angled at C. The internal and external bisectors of the angle BAC meet the side BC in the points D and E. Show that AB touches the circle described on DE as diameter.

Exercise XCVI.

1. Describe a rhombus with a given angle equal to a given parallelogram containing the same angle.

2. $ABCD$ is a square, P is any point in AD, and CP meets BA produced in Q. Prove that the triangles PQD, APB are equal in area.

3. Two fixed parallel chords of a fixed circle are cut by a third chord so that the rectangles contained by the segments of the parallel chords are equal. Show that the middle point of the third chord lies on a fixed straight line.

4. Having given one side of a triangle, and the centre of the circumscribed circle, determine the locus of the centre of the inscribed circle.

5. The exterior angle CBD of the triangle ABC is bisected by the line BE, which cuts AC produced in E. Show that the square on BE, together with the rectangle AB, BC, is equal to the rectangle AE, EC.

6. Show how to find a mean proportional between the sum and the difference of two given straight lines.

Exercise XCVII.

1. If in a triangle ABC the bisector of the exterior angle at A meets the base BC produced in D, prove that, if AD be equal to AC, the difference of the squares on BD and AB is equal to the rectangle AB, AC.

2. The sum of the squares on the sides of any quadrilateral is equal to the sum of the squares on its two diagonals together with four times the square on the line joining the middle points of the diagonals.

3. Show how to divide a given straight line into two parts, the rectangle contained by which parts shall be equal to the square on a given line.

4. ABC is an equilateral triangle inscribed in a circle. Tangents to the circle at A and B meet in M. Show that a diameter drawn from M, and meeting the circumference in D and C, bisects the angle AMB, and that DC is equal to twice MD.

5. $ABCD$ is a parallelogram whose diagonals AC, BD intersect in Q. L and M are two points on AC and BD respectively, such that LM is parallel to AB. Prove that if AM, BL intersect in R, then QR is parallel to AD.

6. OA, OB are tangents to a circle, and OCD is drawn meeting the circumference in C and D. Join AC, CB, BD, DA. Then shall the rectangle AD, BC be equal to the rectangle AC, BD.

Exercise XCVIII.

1. If from any point within an equilateral triangle perpendiculars be drawn to the three sides, show that their sum is equal to a perpendicular drawn from one of the angles to the opposite side.

2. $ABCD$ is a square. P and R are the middle points of the opposite sides AB and CD. Q and S are any two points in the sides AD and BC respectively. Prove that the area of the quadrilateral $PQRS$ is half that of the square.

3. If two circles intersect, and if from either point of intersection two diameters be drawn, show that the straight line joining the extremities of the diameters will pass through the other point of intersection, and will be perpendicular to the chord of intersection.

4. A circle is described touching the base of a triangle and the other sides produced. If the perimeter of the triangle be equal to the diameter of the circle, show that the triangle is right-angled.

5. Construct a triangle, having given the base, the ratio of the sides and the vertical angle.

6. In a given circle inscribe a triangle of given area, having its vertex at a fixed point in the circumference, and its vertical angle equal to a given rectilineal angle.

Exercise XCIX.

1. $ABCD$ is a parallelogram, and a straight line, drawn parallel to AB, meets AD in P, AC in Q, and BC in R. Prove that the triangle APR is equal to the triangle AQD.

2. On the same side of the straight line ABC equal rectangles $ABDE$, $ABFG$ are described. Prove that BG is parallel to DF.

3. From a point C, whose distance from the centre O of a circle is equal to the diameter of the circle, tangents CD, CA are drawn to the circle, and B is the point where CO produced meets the circle again. Prove that $ABDC$ is a rhombus.

4. In a given triangle inscribe a rhombus, one of whose sides shall coincide in direction with a side of the triangle, and one of whose angles shall be at a given point in that side.

5. Inscribe in a given triangle a parallelogram similar to a given parallelogram.

6. If AD is the perpendicular from A on the opposite side BC of a triangle, and O is the point where the three perpendiculars drawn to the sides from the opposite angles intersect, prove that the rectangle BD, DC is equal to the rectangle DO, DA.

Exercise C.

1. ABC is an equilateral triangle. P is a point in BC. Find a point Q in CA, or CA produced, such that the triangles ABP, PCQ may be equal.

2. Divide a given straight line so that the rectangle contained by the whole and one part may be three times the square on the other part.

3. AOB is the diameter of a circle, BPC and APD are chords of the circle intersecting in P, and they are such that DC subtends a right angle at the centre of the circle. Prove that the triangles APC, BPD are isosceles.

4. If an equilateral triangle be inscribed in a circle, and the adjacent arcs, cut off by two of its sides, be bisected, prove that the line joining the points of bisection will be trisected by the sides.

5. Two sides of a triangle, whose perimeter is constant, are given in position. Show that the third side always touches a certain circle.

6. Through the vertices B and C of a triangle ABC, straight lines BL, CM are drawn parallel to one another to meet any straight line through A in L and M respectively Prove that if LO be drawn parallel to AC to meet BC in O, then OM is parallel to AB.

Exercise CI.

1. The opposite sides of a hexagon $ABCDEF$ are equal and parallel. Show that the triangles ACE, BDF are of equal area.

2. Construct an isosceles right-angled triangle which shall be double of a given rectilineal figure.

3. A circle is described to pass through a fixed point A, and also through the extremities P, P' of any diameter of a circle, whose centre is O. Prove that it will cut AO produced in a fixed point.

4. An equilateral triangle ABC is inscribed in a circle, and from A a straight line is drawn cutting BC, and meeting the circle again in D. Prove that AD is equal to the sum of BD and DC.

5. ABC is a triangle inscribed in a circle, and perpendiculars are drawn from any point in the circumference to the sides of the triangle. Prove that the points in which they meet the sides are in one straight line.

6. If four points A, B, C, D be taken in a straight line so that the rectangle contained by AD, BC is equal to the rectangle contained by AB, CD, and a point P be taken on the circumference of a circle described on BD as diameter, and this point be joined to A, B and C, prove that BP bisects the angle APC.

Exercise CII.

1. In a triangle ABC, D is the middle point of the base BC. BE is drawn perpendicular to the line bisecting the angle A, and DE is produced to F, so that EF is equal to DE. Prove that DF is equal to the difference of the sides AB, AC.

2. If $ABCD$ be a straight line bisected in B and divided in C so that the square on BC is equal to the rectangle BD, CD, show that three times the square on BC together with the sum of the rectangles AC, CD and BC, CD is equal to twice the square on AB.

3. AB is a common chord of the segments ACB, $ADEB$ of two circles, and through C, any point in ACB, are drawn the straight lines ACE, BCD. Prove that the arc DE is invariable.

4. A circle B passes through the centre of another circle A. a triangle is described circumscribing A and having two of its angular points on B. Prove that the third angular point is on the line joining the centres of A and B.

5. AB and CD are two parallel straight lines. AC, BD meet in E, and AD, BC meet in F. Show that EF bisects AB and CD.

6. In a right-angled triangle ABC, BD is drawn perpendicular to the hypotenuse AC, and in BD produced a point E is taken so that DE is a third proportional to BD and CD. Prove that the triangles AEC, BEC are equal.

Exercise CIII.

1. If the sides AB, BC, CA of a triangle ABC be respectively bisected in R, P, Q, and AP, CR intersect in O, then, if OQ, OB be joined, show that QOB is a straight line.

2. On AB a square $ABCD$ is described, and the angles ACE, ACF are made each equal to half the angle of an equilateral triangle, thus inscribing an equilateral triangle CEF in the square. Prove that AB is divided in E, so that the square on one part is double of the rectangle contained by the whole and the other part.

3. On any two straight lines AB, AC, intersecting at A, semicircles AEB, AEC are described, intersecting at E. Show that the points B, E and C are in one straight line.

4. In the regular pentagon $ABCDE$, the diagonals AC and BE intersect in O. Show that the circle passing through the points C, O, E, touches the sides of the pentagon BC, AE, in C and E respectively.

5. ABC is a triangle, and through D, any point in AB, DE is drawn parallel to BC, meeting AC in E, and CF is drawn parallel to BE, meeting AB produced in F. Prove that AB is a mean proportional between AD and AF.

6. In a straight line given in position determine a point, at which two straight lines, drawn from given points on the same side of the given line, will contain the greatest angle.

Exercise CIV.

1. Draw from a given point in the base of a triangle two straight lines, which shall trisect the triangle.

2. A straight line is divided so that the sum of the squares on the whole line and one part is equal to three times the square on the other part. Prove that the rectangle contained by the whole line and the first part is equal to the square on the second part.

3. Prove that the rectangle contained by two parallel chords AB, DC of a circle $ABCD$ is equal to the difference of the squares on AC and AD.

4. If all the diagonals of a regular pentagon inscribed in a circle be drawn, show that every triangle formed is isosceles, and has its vertical angle either half or three times the angle at the base.

5. If squares $AGFB$ and $AHKC$ be described externally on the sides of a triangle ABC, right-angled at A, and BK, CF be joined, meeting the sides of the triangle in L and M respectively, prove that AL and AM are equal.

6. The diagonals of a quadrilateral $ABCD$ inscribed in a circle intersect in E. If the quadrilateral be such that the rectangle contained by AE and AC is equal to that contained by AB and AD, prove that either AB is equal to AD, or that AC bisects the angle BAD.

Exercise CV.

1. If a straight line DME be drawn through the middle point M of the base BC of a triangle ABC, so as to cut off equal parts AD, AE from the sides AB, AC, produced if necessary, respectively, then shall BD be equal to CE.

2. In any quadrilateral show that the squares on the two sides, which subtend an obtuse angle at the point of intersection of the diagonals, are together greater than the sum of the squares on the two sides which subtend an acute angle at the same point.

3. On the same side of portions AB, AC of a straight line ABC similar segments of circles are described. Prove that the circles touch each other.

4. If in a triangle ABC the straight lines drawn from B and C perpendicular to the opposite sides meet in L, and B', C' be the centres of the circles circumscribing the triangles CLA, ALB respectively, $B'C'$ will be equal and parallel to BC.

5. From the obtuse angle of a triangle draw a straight line to the base, which shall be a mean proportional between the segments, into which it divides the base.

6. Describe a parallelogram equal to and equiangular with a given parallelogram, and having a given altitude.

Exercise CVI.

1. Construct a triangle whose angles shall be equal to those of a given triangle, but whose area shall be four times as great.

2. If ABC be a triangle, whose angle A is a right angle, and BE, CF be drawn bisecting the opposite sides respectively, show that four times the sum of the squares on BE and CF is equal to five times the square on BC.

3. Find the centre of a circle cutting off three equal chords from the sides of a given triangle.

4. The circle inscribed in the triangle ABC touches BC in D. The circle, touching BC and the other sides produced, touches BC in E. Show that D is equal to the difference between AB and AC.

5. Any two diagonals of a regular pentagon cut one another so that the rectangle contained by the whole diagonal and one of the parts, into which it is divided, is equal to the square on the other part.

6. If two triangles have a common angle, show that the areas of the triangles are proportional to the rectangles contained by the sides of the triangles about the common angle.

Exercise CVII.

1. Three straight lines OA, OB, OC are drawn so that the angles AOB, BOC are equal, and from any point D perpendiculars DA, DB, DC are drawn on these straight lines. Prove that the straight lines AB, BC are equal.

2. $ABCD$ is a parallelogram, BOD one of its diagonals, and EOG, FOH are drawn parallel to BC, CD respectively, so that E, F, G, H lie, correspondingly, on the sides AB, BC, CD, DA. If DF, BH be drawn intersecting EG in K, L respectively, prove that OK is equal to OL.

3. If two circles cut one another, find the locus of points from which the tangents drawn to the two circles are equal to one another.

4. Show that the diameter of the circle, which is described about an isosceles triangle, having its vertical angle double of either of the angles at the base, is equal to the base of the triangle.

5. A tangent TP is drawn to a circle from an external point T, and PM is drawn perpendicular to the line which joins T to the centre of the circle. If Q be any point on the circumference of the circle, prove that TQ is to QM in the ratio of TP to PM.

6. A line $ACBD$ is divided, so that AC is to CB as AD is to DB. Show that a semicircle, described on CD, is the locus of B, such that AP is to PB as AC is to CB.

Exercise CVIII.

1. If K be the common angular point of the parallelograms about AC, a diameter of a given parallelogram, and BD be the other diameter, show that the difference of the parallelograms about AC is equal to twice the triangle BKD.

2. Construct a triangle having given the base, the difference of the sides and one of the angles at the base.

3. Through any point C in the common chord of two intersecting circles a straight line $ABCDE$ is drawn, cutting the circles respectively in A, D and B, E. Prove that the rectangle AC, CD is equal to the rectangle BC, CE.

4. Two equilateral triangles are described about the same circle. Show that their intersections will form a hexagon, equilateral but not always equiangular.

5. AD is the perpendicular drawn from the right angle BAC to meet the base BC of the triangle ABC in D. Perpendiculars DE, DF are drawn to the sides AB, AC. Show that a circle will pass through the points B, E, F, C.

6. Describe an equilateral triangle equal in area to two given equilateral triangles.

Exercise CIX.

1. BAC is a right-angled triangle, A being the right angle. $ACDE$, $BCFG$ are the squares on AC and BC. AC produced meets DF in K. Prove that DF is bisected in K, and that AB is double of CK.

2. Given two sides and the angle opposite one of them, construct the triangle.

3. AOB, COD are two chords of a circle which intersect, within the circle, at the point O. Through A a straight line AF is drawn to meet the tangent to the circle at the point C, so that the angle AFC is equal to the angle BOC. Prove that OF is parallel to BC.

4. If the sides of an equilateral and equiangular pentagon be produced to meet, show that the angles formed by these lines are together equal to two right angles.

5. If upon the diagonals AC, BD of a quadrilateral $ABCD$, whose sides AB, CD are parallel, parallelograms be constructed whose sides opposite AC, BD intersect on AB, show that the sum of these two parallelograms is double of the triangle ABC.

6. Through F, a point on the diagonal BD of a rectangle $ABCD$, are drawn two straight lines EFG, KFL parallel to the sides of the rectangle and intersecting them in E, G, K, L respectively. Prove that the rectangle BF, FD is equal to the sum of the rectangles EF, FG and LF, LK.

Exercise CX.

1. If ABC be a triangle, in which C is a right angle, show how, by means of Euclid, Book I., to draw a straight line parallel to a given straight line so as to be terminated by CA and by CB produced, and bisected by AB.

2. If the base BC of a triangle ABC be trisected in the points D and E, and the lines AD, AE be drawn, prove that the sum of the squares on AB and AC exceeds the sum of the squares on AD and AE by four times the square on DE.

3. AB is the chord of a segment ACB of a circle. C is any point in the arc, AC is produced to P, so that PC is equal to CB. Prove that the point P lies on the circumference of a certain fixed circle.

4. Having given the base and the vertical angle of a triangle, prove that the straight line bisecting the vertical angle passes through a fixed point.

5. If to the circle, circumscribing the triangle ABC, a tangent at C be drawn, cutting AB produced in D, show that AD is to DB in the duplicate ratio of AC to CB.

6. If ABC be a right-angled triangle, and EF, parallel to the hypotenuse BC, meet AB, AC in E, F, then EH, FL, AK being drawn perpendicular to BC, show that the difference of the rectangles CK, CH and BL, BK is equal to the difference of the squares on AB, AC.

Exercise CXI.

1. ABC, DBC are triangles on the same base and between the same parallels. $ABCK$ is a parallelogram, and AC, BD meet in O. Show that the difference between the triangles BOC, AOD is equal to the triangle DCK.

2. Given the sum and the sum of the squares on two straight lines, find them.

3. $ABCD$ is a parallelogram. A circle through the points A, B cuts AD, BC in M, N respectively. Show that $KLMN$ is also a parallelogram.

4. A triangle ABC is inscribed in a circle, and DF is the diameter perpendicular to BC. Prove that the difference of the angles at B and C is either double the angle AFD, or is the supplement of double the angle AFD.

5. If perpendiculars be drawn from the extremities of a diameter of a circle upon any chord or any chord produced, the rectangle contained by the perpendiculars is equal to the rectangle contained by the segments between the feet of the perpendiculars and either extremity of the chord.

6. Circles are described on the two sides of a right-angled triangle, which contain the right angle. Show that the square on their common tangent is equal to the area of the triangle.

Exercise CXII.

1. Let *ABC*, *ABD* be two equal triangles, upon the same base *AB*, and on opposite sides of it. Join *CD*, meeting *AB* in *E*. Show that *CE* is equal to *ED*.

2. *A* square *ABCD* and an equilateral triangle *ABE* are described on the same base *AB* and on opposite sides of it. If *F*, the middle point of *AE*, be joined with *C*, and *AE* be produced to *G*, so that *EG* is equal to *FB*, show that *GB* and *FC* will be equal.

3. *P* is any point on the circumference of a circle which passes through the centre *C* of another circle, and *PQ*, *PR* are the tangents drawn from *P* to the other circle. Show that *CP* and *QR* meet on the line joining the points of intersection of the circles.

4. *AB*, *CD* are chords of a circle intersecting at *O*, and *AC* *DB* meet at *P*. If circles be described about the triangles *AOC*, *BOD*, show that the angle between their tangents at *O* will be equal to the angle *APB*, and that their other common point will lie on *OP*.

5. *AB* is the diameter of a circle. *PM* is drawn from *P*, a point in the circumference at right angles to *AB* and meeting it in *M*. *AQ* is the tangent at *A*. If the line joining *BQ* bisects *PM*, show that *QP* touches the circle.

6. *ABCD* is a quadrilateral inscribed in a circle, and its diagonals intersect in *F*. Prove that the rectangle *AF*, *FD* is to the rectangle *BF*, *FC* as the square on *AD* is to the square on *BC*.

Exercise CXIII.

1. The side of the parallelogram $ABCD$ is produced to E. Find a point in BC, such that the triangle AFE may be equal to half the parallelogram $ABCD$.

2. ABC is a right-angled triangle, having A as the right angle. If $ABFG$ and $ACKH$ be the squares on AB and AC, and if M and N be the feet of the perpendiculars dropped from F and K respectively on BC produced, prove that BM is equal to CN.

3. From a point P outside a circle, a perpendicular PN is drawn to a diameter AB, such that AN is equal to the tangent from P. BA is produced to Q, so that AQ is equal to AN. If a circle be described on BQ as diameter, and a perpendicular to AB be drawn from A, cutting this circle in K, prove that AK and PN are equal.

4. If AD, BE, CF be the perpendiculars let fall from the angular points of a triangle ABC on the opposite sides, show that DEF is the triangle of least perimeter which can be inscribed in the triangle ABC.

5. On the side AB of an equilateral triangle ABC a square $ABDE$ is described. Through A the line AF is drawn parallel to BC, to meet DE in F, and CA is produced to meet DE produced in G. Prove that AFG is an equilateral triangle.

6. From D, a point in CA, the base of an isosceles triangle BCA, lines DE, DF, are drawn to the equal sides BC, BA, such that the angles CDE, ADF are equal. If AE, CF be drawn, show that the triangles AED, CDF are equal.

Exercise CXIV.

1. Straight lines AD, BE, CF are drawn within the triangle ABC, making the angles DAB, EBC, FCA all equal to one another. If the lines AD, BE, CF do not meet in a point, prove that the angles of the triangle formed by them are equal to those of the triangle ABC, each to each.

2. P is any point in the side AC of a triangle ABC. Find a point Q in CB produced such that PQ may be bisected by AB.

3. Through one of the points of intersection of two circles, of which the centres are A and B, a chord is drawn meeting the circles in P and Q respectively. The lines PA, QB intersect in C. Find the locus of C.

4. P is a point on the circumference of the circle circumscribing a given triangle ABC. The sides of a triangle DEF are parallel to the straight lines PA, PB, PC Show that the triangle DEF is equiangular to the triangle ABC.

5. Describe an isosceles triangle equal in area to a given triangle, and having its vertical angle equal to an angle of the given triangle.

6. Two equal circles have their centres at A and B. O is a fixed point outside those circles. A is the centre of a third circle, whose radius is equal to OB. Prove that the tangents from O to the three circles are proportional to the sides of a right-angled triangle.

Exercise CXV.

1. ABC is a triangle, and from A a line AD is drawn to the base, making the angle BAD equal to the angle ACB. A second line is drawn to meet the base in E, so that AE is equal to AD. Show that the angle CAE is equal to the angle ABC.

2. Describe a right-angled triangle equal to a given rectilineal figure, and such that one of the sides containing the right angle is double of the other.

3. ACB, ADB are two segments of circles on the same base AB. If through any point C in the arc ACB, two straight lines ACD, BCE be drawn to meet the arc ADB in D and E, prove that the arc DE is of constant length.

4. From each angular point of a triangle a perpendicular is let fall on the opposite side. Prove that the distance of the point of intersection of these perpendiculars from any one of the angular points is equal to twice the distance of the centre of the circumscribed circle from the side opposite to that angular point.

5. The base AB of an isosceles triangle ABC is produced both ways to D and E, so that the rectangle AD, BE is equal to the square on AC. Show that the triangles DAC, EBC are similar.

6. If AC be drawn from A to a point C in the base of the triangle ABD, so that ABD, ACD are similar triangles, show that DA touches the circle described about ABC.

Exercise CXVI.

1. ABC is an isosceles triangle whose vertex is A, and points P, Q are taken in AB, AC respectively, so that the sum of AP and AQ is equal to the sum of AB and AC. Prove that the middle point of PQ lies on BC.

2. On each of two sides of an equilateral triangle a parallelogram is described. Show how to apply to the third side a parallelogram whose area is equal to the sum of the areas of these two parallelograms.

3. ANX is a tangent to a circle AMP. P is any point on the circumference, and M is the middle point of either of the segments into which the circle is divided by the points A and P. PM is joined and produced to cut ANX in N. Prove that the angle PNX is three times the angle MAX.

4. $ABCD$ is a quadrilateral figure inscribed in a circle, and its diagonals intersect in O. About the triangle AOB a circle is described. Prove that the straight line touching this circle at the point O is parallel to one of the sides of the figure $ABCD$.

5. Divide a circle into two segments, such that the angle in one of the segments shall be double of the angle in the other segment.

6. From a point in the base of a triangle parallels are drawn to the other sides. Find a second point in the base, from which parallels drawn similarly will construct a parallelogram equal to that first drawn.

Exercise CXVII.

1. On a given straight line describe a triangle, having one of its angles adjacent to this side equal to a given angle, and having the sum of its other two sides equal to another given straight line.

2. Divide a given straight line into two parts, such that if a right-angled triangle be formed, having these two parts for the two sides containing the right angle, the hypotenuse may be the least possible.

3. From a point O are drawn two straight lines, OT to touch a given circle at T, and OC to pass through its centre C; and TN is drawn to cut OC at right angles in N. Show that the circle, which touches OC at O and passes through T, cuts the given circle in a point S, such that the straight line TS produced bisects NO.

4. The sides AD, BC, and AB, CD of a quadrilateral $ABCD$ inscribed in a circle whose centre is O, meet respectively in E and F. From E a perpendicular EM is let fall on OF. Prove that the rectangle contained by OF, OM is equal to the square on the radius of the circle.

5. If from any point in the circumference of a circle any number of chords be drawn, show that the locus of their points of bisection will be a circle.

6. If $ABCD$ be any quadrilateral figure inscribed in a circle, and BK, DL be perpendiculars on the diagonal AC, show that BK is to DL as the rectangle AB, BC is to the rectangle AD, DC.

Exercise CXVIII.

1. Two triangles ABC, ABD are on the same base AB and between the same parallels, and the distances between the vertices C, D is half of the common base. If AD, BC meet in E, and AC, BD when produced meet in F, prove that the quadrilateral $CEDF$ is equal to the triangle AEB.

2. If each of the diagonals of a quadrilateral divide it into two triangles which are equal in area, prove that the quadrilateral is a parallelogram.

3. A straight line intersects one circle in P, Q, and a second circle in R, S. If the tangents at P and R are parallel, show that the tangents at Q and S are also parallel.

4. Prove that the area of a regular polygon of twelve sides is equal to three times that of a square described on the radius of the circle circumscribing the polygon.

5. Prove that, if from the vertex of a triangle perpendiculars be drawn to the external bisectors of the angles at the base, the line joining the feet of these perpendiculars will be parallel to the base.

6. AB is a diameter and P any point in the circumference of a circle. AP and BP are joined and produced, if necessary. If from any point C in AB a perpendicular be drawn to AB, meeting AP and BP in D and E respectively, and the circumference of the circle in F, show that CD is a third proportional to CE and CF.

Exercise CXIX.

1. If the base AB of a triangle ABC be produced towards B, and at A a line is drawn making with AC an angle, on the opposite side to AB, such that this angle exceeds or falls short of the exterior angle at B by as much as the angle at B of the triangle exceeds or falls short of the angle A, then shall the line at A be in one and the same straight line with the base AB.

2. Divide a straight line into two parts so that the sum of the squares on the whole line and one part shall be equal to three times the square on the other part.

3. Two circles touch at A. Prove that if the tangents drawn to the circles from a point P be equal, P must be in the tangent at A.

4. If from the angular point C of a rectangle $ABCD$ a line FCE be drawn at right angles to the diagonal AC, meeting AB, AD produced in E, F respectively, a circle can be described about the figure $BDEF$.

5. If the bisectors of the external and internal angles at the vertex A of a triangle ABC meet the base in the points D, E respectively, show that A is a point on the circumference of a circle whose diameter is DE.

6. Show that the three lines, joining the angular points of a triangle to the centre of the circle circumscribing the triangle, are respectively perpendicular to the three straight lines, which join the feet of the perpendiculars drawn from the angular points upon the opposite sides.

Exercise CXX.

1. The diagonals of a quadrilateral *ABCD* intersect in *O*, and the parallelograms *OAEB*, *OBFC*, *OCGD*, *ODHA* are completed. Prove that *EFGH* will be a parallelogram, and will be double of *ABCD*.

2. A straight line *AB* is produced both ways to *C* and *D*, so that *BD* is twice *AC*. Show how to find the points *C* and *D* when the rectangle *CA*, *AD* is equal to the square on *AB*.

3. Through any two points *A*, *B* in the straight line *ABC* any number of circles are drawn. From any point *C* in the line a tangent is drawn to each circle. Prove that the points of contact of the tangents all lie in the circumference of a circle whose centre is *C*.

4. *CD* is drawn from the angle *C*, perpendicular to the hypotenuse *AB* of a right-angled triangle *ABC*. *DE*, *DF* are drawn parallel to *AC*, *BC*, meeting *BC*, *AC* in *E*, *F* respectively. Show that a circle can be described about the figure *ABEF*.

5. On a given base construct a triangle, whose sides shall be in the ratio of two to one, and whose vertical angle shall be two-thirds of a right angle.

6. Through a given point *E*, in the side *AB* of a triangle *ABC*, draw a straight line cutting *AC* in *F* and *BC* produced in *D*, so that *EF* may be to *FD* as *FC* to *FA*.

Exercise CXXI.

1. Three straight lines meet at a point O, and P is a given point in any one of them. Through P draw a line to meet the other lines at Q and R, and so as to make the triangles OPQ, OPR equal to one another.

2. Divide a given straight line into two parts, so that the rectangle contained by the whole and one part may be equal to the rectangle contained by the other part and another given straight line.

3. If the exterior angles of any quadrilateral be bisected by four straight lines, prove that the quadrilateral formed by the intersection of these lines can have a circle, described about it.

4. If AB, AC are the tangents at the points B, C of a circle, and if D is the middle point of the arc BC, prove that D is the centre of the circle inscribed in the triangle ABC.

5. If two triangles have equal vertical angles, and if they be also equal in area, the rectangle contained by the sides about the vertical angle in one shall be equal to the rectangle contained by the sides about the vertical angle in the other.

6. A straight line is drawn parallel to the parallel sides of a trapezium and terminated by the other sides so as to divide a diagonal into segments respectively proportional to the two parallel sides which meet them. Show that the line is bisected by the diagonal.

Exercise CXXII.

1. If points E, F be taken on the sides CA, AB of a triangle ABC, such that CE, AF are the third parts of CA, AB respectively, and if O be the point of intersection of BE, CF, prove that EO is the seventh part of EB.

2. If the base BC of a triangle ABC be trisected in the points D and E, prove that the sum of the squares on AB, AC is equal to the sum of the squares on AD, AE and BE.

3. Two circles, whose centres are A and B, intersect. P is a point on one, and a chord through P meets the other in Q and R. PM drawn perpendicular to the common chord meets it in M. Show that the rectangle contained by PM and AB is half of that contained by PQ and PR.

4. If two triangles, having the angles of the one equal to those of the other, each to each, be circumscribed round the same circle, show that a circle will pass through any two corresponding angular points and the intersections of the lines containing those angles.

5. ABC is a triangle, and lines are drawn through B and C to meet the opposite sides in E and F. If BE, CF intersect in a point on the line joining A to the middle point of BC, show that EF is parallel to BC.

6. Two circles A and B touch another circle C internally, and a common tangent to the circles A and B meets the circle C in R and S. Prove that the rectangles under RP, SP, and RQ, SQ are in the ratio of the radii of A and B.

Exercise CXXIII.

1. Four points lie in a plane, no one being within the triangle formed by joining the other three. Determine the point, the sum of whose distances from these four points is the least possible.

2. Prove that if a point be such that the difference of the squares described on the lines joining it with two other points in the same plane with it is a given magnitude, the point must lie on one of two straight lines.

3. ABC is a triangle. A circle, whose centre is A and radius AB, cuts BC in E. Show that the rectangle contained by CE, CB is equal to the difference of the squares on CA and BA.

4. If O be the centre of the circle inscribed in a triangle ABC, and AO, BO be produced to meet the opposite sides in E and F, prove that, if a circle can be described round the quadrilateral $CEOF$, the angle C must be equal to the third part of two right angles.

5. The point O is a fixed point, and AB is a fixed straight line. If any point P be taken in AB, and if in the straight line OP a point Q be taken such that the rectangle contained by OQ and OP is constant, find the locus of the point Q.

6. A and B are fixed points, and AC, AD are fixed straight lines, such that BA bisects the angle CAD. If any circle passing through A and B cut off the chords AK and AL from AC and AD, prove that the sum of the lengths of AK and AL is always the same.

Exercise CXXIV.

1. Any parallelograms $ABDE$, $ACFG$ are described externally on the sides AB, AC of any triangle ABC. If DE, FG be produced to meet in L, and BM, CN be drawn parallel and equal to LA, show that the parallelogram $BMNC$ is equal to the sum of the parallelograms $ABDE$ and $ACFG$.

2. If points F, D be taken in the sides AB, BC respectively of a triangle ABC, so that AF is the fourth part of AB, and CD the third part of CB, and if AD, CF intersect in O, prove that AD is bisected at the point O.

3. Two equal circles touch externally. Through the point of contact chords, one to each circle, are drawn at right angles to each other. Prove that the line joining the extremities of the chords is parallel to the straight line joining the centres of the circles.

4. If the diagonals AC, BD of a quadrilateral $ABCD$ intersect in E, prove that the centres of the circles described about the triangles EAB, EBC, ECD, EDA are the angular points of a parallelogram.

5. C being the obtuse angle of a triangle ABC, and D, E the feet of the perpendiculars drawn from A and B respectively to the opposite sides produced, prove that the square on AB is equal to the sum of the rectangles contained by BC, BD and AC, AE.

6. Determine two lines such that the sum of their squares may be equal to a given square; and the rectangle contained by them equal to a given rectangle.

Exercise CXXV.

1. Parallelograms $AFGC$, $CBKH$ are described on AC, BC, outside the triangle ABC. FG, KH meet in Z. ZC is joined, and through A and B lines AD, BE are drawn, both parallel to ZC, and meeting FG, KH in D and E respectively. Prove that $ADEB$ is a parallelogram, and is equal to the sum of the parallelograms FC, CK.

2. ABC is an isosceles triangle, of which A is the vertex. AB, AC are bisected in D and E respectively; BE, CD intersect in F. Show that the triangle ADE is equal to three times the triangle DEF.

3. ACB, DCE are two diameters of a circle at right angles to each other. A circle, with centre O, touches this circle internally at P, and also touches AB in Q. Prove that PO passes through C, and that PQ passes through D or E.

4. A hexagon inscribed in a circle has a pair of opposite sides parallel to one another. Prove that it has a pair of opposite angles equal to one another.

5. If ABC be a right-angled triangle, and D any point in the hypotenuse AB, find the point P to which AB must be produced, so that PA shall be to PB as AD to DB.

6. Two equal parallelograms $ABCD$ and $AEFG$ have a common angle A; AB, AE being in the same straight line, and AD, AG in the same straight line. Prove that BG, CF, DE are all parallel, and that one of these lines is equal to the sum of the other two.

Exercise CXXVI.

1. Three parallelograms are described having the sides of a given triangle for diagonals, and one angular point common. Show that the remaining points are the angular points of a triangle equal to the original triangle.

2. If BAC be a right-angled triangle, A being the right angle, and $ACDE$ be the square on AC, and $BCFG$ the square on BC, show that if AC be produced, it will bisect the line DF.

3. If two circles intersect each other, prove that each of the common tangents subtends supplementary angles at the points of intersection.

4. Show how to inscribe a square in a given semicircle (1) without using the Sixth Book of Euclid, and (2) by means of the Sixth Book.

5. AD is drawn bisecting the angle BAC of the triangle BAC, and meeting BC in D. FDE is drawn perpendicular to AD, to meet AB and AC, produced if necessary, in F and E respectively, and EG is drawn parallel to BC, meeting AB in G. Prove that BG is equal to EF.

6. Draw a line parallel to the base of a triangle, so as to divide the triangle into two parts, which shall be in a given ratio.

Exercise CXXVII

1. In the base BC of a triangle ABC any point D is taken. Draw a straight line such that if the triangle ABC be folded along this straight line, the point A shall fall on the point D.

2. The diagonals of a rhombus $ABCD$ intersect in E, and on one of them is taken a point P. Prove that the squares on PA, PB, PC, PD are together equal to four times the square on PE together with twice the square on a side of the rhombus.

3. Three circles are drawn so that each touches the other two externally, and common tangents are drawn to the circles at their points of contact. Prove that these common tangents meet in a point.

4. In a given circle inscribe a triangle of given area, having its vertex at a fixed point in the circumference, and its vertical angle equal to a given rectilineal angle.

5. AOB, COD are two intersecting straight lines, such that the rectangle contained by AO, OD is equal to that contained by BO, OC. Prove that if parallelograms be constructed on AO, OC and BO, OD as adjacent sides respectively, the diagonals which pass through O are in the same straight line.

6. Show that if a quadrilateral $ABCD$, all of whose sides are unequal, be inscribed in a circle, of which BD is a diameter, the ratio of AB to CD cannot be equal to the ratio of BC to AD.

Exercise CXXVIII.

1. $ABCD$ is a parallelogram, E is the middle point of BC, and AE and DC produced meet in F. Prove that the triangle DEF is half of the parallelogram $ABCD$.

2. Prove that if a point lie on a circle having its centre at the middle point of the line joining two given points, the sum of the squares described on the lines joining it with the given points is constant.

3. Two circles intersect in P and Q. Any line through P cuts the circles in R and S. Show that the angle RQS is constant.

4. If a point P be taken on the side BC of a triangle ABC, and if circles be described about the triangles ABP, ACP, prove that the angle between the straight lines, which touch these circles at P, is independent of the position of P in BC.

5. If ABC be a triangle right-angled at C, and if the line AD bisecting the exterior angle at A meet the base BC produced in D, prove that the sum of AB, AC is a mean proportional between BC and the sum of BD, CD.

6. Describe an equilateral triangle, whose area shall be equal to that of a given square.

Exercise CXXIX.

1. $ABCD$ is a square, and E a point in BC. A straight line EF is drawn at right angles to AE, and meets the straight line, bisecting the angle between CD and BC produced, in F. Prove that AE is equal to EF.

2. $ABCDE$ is a straight line, C being the middle point of BD. Prove that the square on AC together with the rectangle BE, DE is equal to the square on EC together with the rectangle AB, AD.

3. With three given points, not in the same straight line, as centres, describe three circles, each of which shall touch the other two.

4. ABC is a right-angled triangle, A being the right angle. Prove that the hypotenuse BC is equal to the difference between the radius of the inscribed circle of the triangle, and the radius of the circle which touches BC and the other two sides produced.

5. On a given base construct a triangle whose sides shall be in the ratio of three to one, and whose vertical angle shall be half a right angle.

6. ABC is a triangle, and AM the perpendicular upon BC, and P is any point in BC. If O, O' be the centres of the circles described about ABP, ACP, show that the rectangle AP, BC is double of the rectangle AM, OO'.

Exercise CXXX.

1. Each of the equal sides of an isosceles triangle is greater than the third side. Prove that the angle contained by the equal sides is less than an angle of an equilateral triangle.

2. On the side BC of any triangle ABC, and on the side of BC remote from A, a square $BDEC$ is described. Prove that the difference of the squares described on AB and AC is equal to the difference of the squares described on AD and AE.

3. The centre C of a circle BPQ lies on another circle APQ. PBA is a diameter of the circle APQ. Prove that PC and BQ are parallel.

4. ABC is a triangle inscribed in a circle. Through each of the angular points two straight lines are drawn, parallel to the lines joining the centre of the circle to the other angles of the triangle. Prove that these lines will form an equilateral hexagon, and that each of the angles of this hexagon is equal to one of its other angles.

5. Through a fixed point A, on the circumference of a circle, a chord AB is drawn, and produced to a point M, so that the rectangle contained by AB and AM is constant. Find the locus of M.

6. ABD is a triangle, AB is produced to E, AD is a line meeting BC in D, BF is parallel to ED, and meets AD in F. Determine a triangle similar to ABC and equal to AEF.

Exercise CXXXI.

1. O, O' are points within a triangle ABC, such that OA, $O'A$ are equally inclined to the sides AB and AC respectively and OB, $O'B$ equally inclined to the sides BA and BC respectively. Prove that OC, $O'C$ are equally inclined to the sides CA and CB respectively.

2. If the straight line AB be bisected in C and produced to D, so that the square on BD is equal to twice the square on BC, show that the rectangle contained by AD, DB will be equal to the rectangle contained by AB, CD.

3. A point of intersection of two circles is joined to the middle point of the line joining their centres. Prove that the chords from the point of intersection of the circles at right angles to the line so drawn are equal.

4. $ABCD$ is a quadrilateral inscribed in a circle. AB, DC produced meet in E, and a circle is described round the triangle AED. Show that the tangent to this circle at E is parallel to BC.

5. Through one of the points of intersection of two given circles draw a straight line, such that the parts of it intercepted by the two circles are in a given ratio.

6. AB, AC are two straight lines; B and C are given points in them. BD is drawn perpendicular to AC, and DE perpendicular to AB. In like manner CF is drawn perpendicular to AB, and FG to AC. Show that EG is parallel to BC.

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